



Thermodynamics "B"

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Thermo-dynamics "B"

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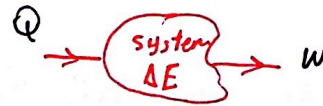
[1]

second law of Thermodynamics :->

The 1st law of Thermodynamics \Rightarrow is energy balance law between the system and its surrounding, it concerns only with the exchanged amount of energy between the system & its surrounding.

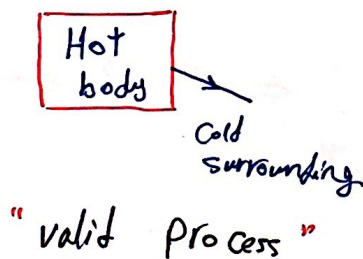
$$\Rightarrow Q - W = \Delta E_{\text{system}}$$

$$\Rightarrow Q_{1 \rightarrow 2} - W_{1 \rightarrow 2} = E_2 - E_1$$

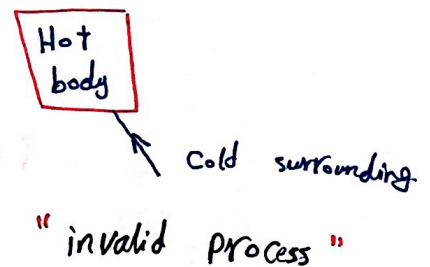


it doesn't concern with the direction of process or the possibility of its occurring.

Ex



&

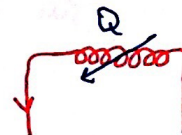


Ex



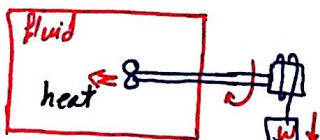
"valid process"

&



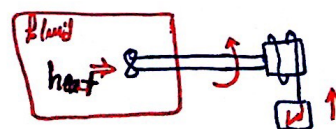
"invalid process"

Ex



direct transformation of work into heat is "simple process"

&



direct transformation of heat into work is "not simple process"

So, the 2nd law:

[2]

- 1- Can identify the process direction.
- 2- it's used also in determination of the Theoretical Limits (max limits) of performance of the engineering systems.

Heat engine

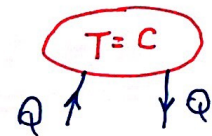
Refrigerator

Thermal energy reservoir \Rightarrow

It is hypothetical body that can absorb or reject heat without any change in its Temperature.

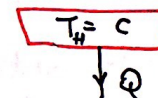
Source \Rightarrow

--- that can supply or reject heat ---



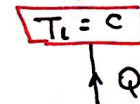
Sink \Rightarrow

--- that can absorb heat ---



EX

Oceans, Seas, rivers, atmospheric air,
air in room, Furnaces.



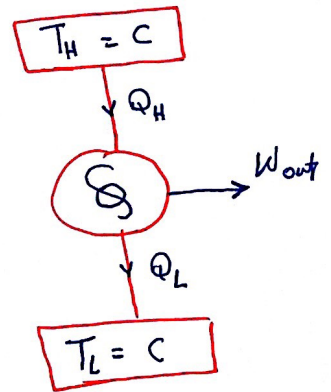


Heat engines \Rightarrow

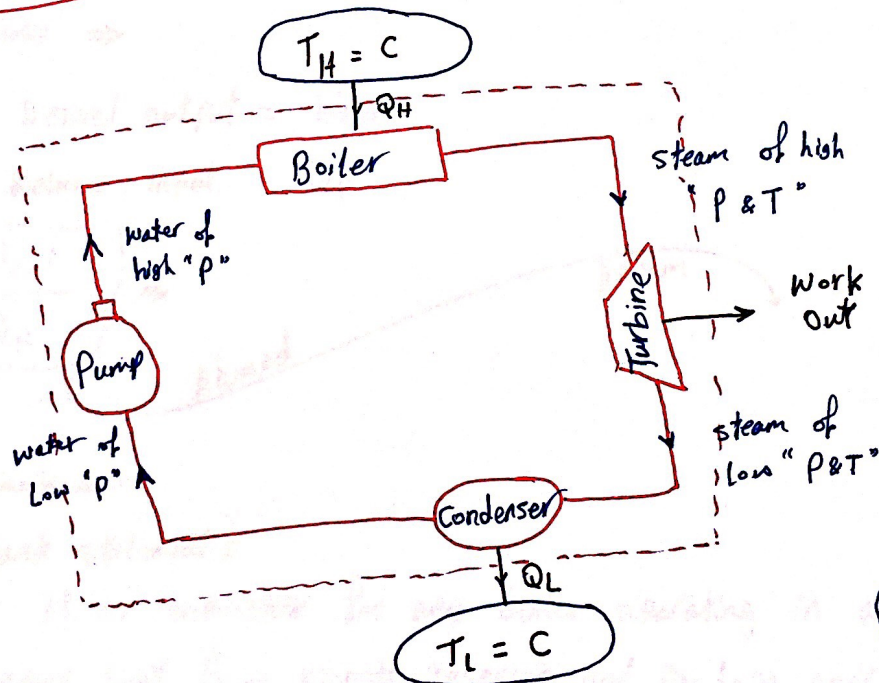
They are devices that transform heat into work.

They are characteristic by \Rightarrow

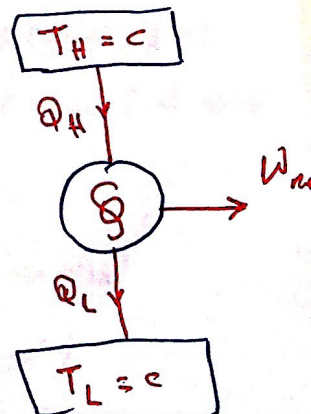
- 1- They receive heat from high Temp source.
- 2- They convert part of This heat into work.
- 3- They reject The remaining part of heat to Low Temp sink.
- 4- They involve working fluid with performance a cycle.



Ex "Steam power plant" is a good example of heat engine \Rightarrow



\Rightarrow The cycle of all represent closed system



$$Q_{\text{net}} - W_{\text{act}} = \Delta E_{\text{cycle}}$$

$$Q_{\text{net}} = W_{\text{net}}$$

$$Q_H - Q_L = W_{\text{net}}$$

$$Q_H = Q_L + W_{\text{net}} \Rightarrow$$

Performance \Rightarrow ratio between desired output and required input.
general $= \frac{\text{Desired Output}}{\text{Required Input}}$

For Heat Engine \Rightarrow

$$\text{Desired output} = W_{\text{net}}$$

$$\text{Required input} = Q_H$$

$$\eta_{\text{th}} = \frac{W_{\text{net}}}{Q_H} \Rightarrow$$

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2nd law of Thermodynamics

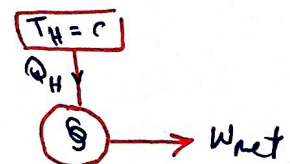
((Kelvin - plank statement))

it is impossible for any device operating in a cycle (H.E) to receive heat from a single reservoir and produce a net amount of work.

$$Q_L = 0$$

$$Q_H = W_{\text{net}}$$

$$\eta_{\text{th}} = \frac{W_{\text{net}}}{Q_H} = 1$$



H.E doesn't satisfy kelvin plank.

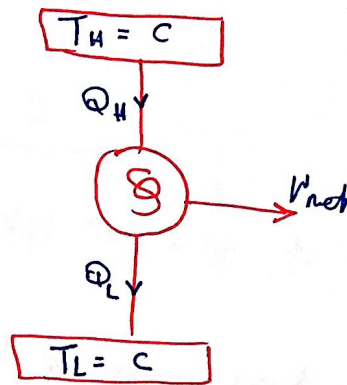


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$$Q_L \neq 0$$

$$Q_H > W_{net}$$

$$\eta_{th} = \frac{W_{net}}{Q_H} < 1$$



H.E satisfy
kelvin plank

Caloric volume.

$$\dot{Q}_H = \dot{m}_p \times C.V$$

$$\frac{kg}{s} \times \frac{kJ}{kg} = kW$$

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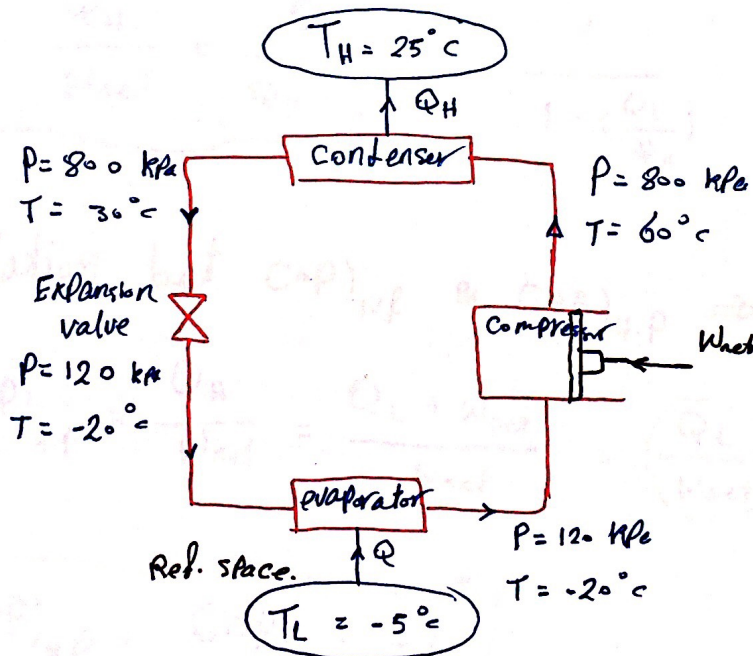
→ Heat is transferred from High Temp body to Lower one.

→ To perform the process in the reverse direction we need special device that is called (Refrigerator or Heat Pump)

Refrigerator → is a cyclic device that transfers heat from low Temp body to the higher one.

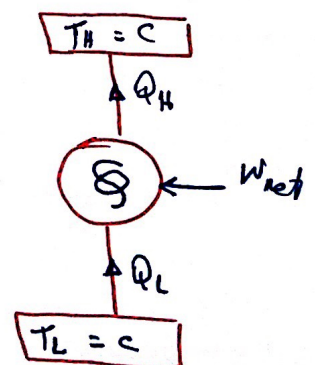
The used working fluid is called "Refrigerant" $R134a$

⇒ Vapor Compression refrigeration cycle: →



→ The cycle as all represent closed system.

$$Q_L + W_{net} = Q_H$$



Refrigerator or H. Pump.



Performance

for Refrigerator

Desired output = Q_L

Required input = W_{net}

$$\text{Coefficient of Performance} \Rightarrow \text{COP}_{ref} = \frac{Q_L}{W_{net}} = \frac{Q_L}{Q_H - Q_L} = \frac{1}{\left(\frac{Q_H}{Q_L}\right) - 1} > 1$$

for Heat Pump

Desired output = Q_H

Required input = W_{net}

$$\text{COP}_{H.P} = \frac{Q_H}{W_{net}} = \frac{Q_H}{Q_H - Q_L} = \frac{1}{1 - \left(\frac{Q_L}{Q_H}\right)} > 1$$

Relation bet COP_{ref} & $\text{COP}_{H.P} \Rightarrow$

$$\text{COP}_{H.P} = \frac{Q_H}{W_{net}} = \frac{Q_L + W_{net}}{W_{net}} = \left(\frac{Q_L}{W_{net}}\right) + 1$$

$$\Rightarrow \text{COP}_{H.P} = \text{COP}_{ref} + 1$$



Second law of Thermodynamics

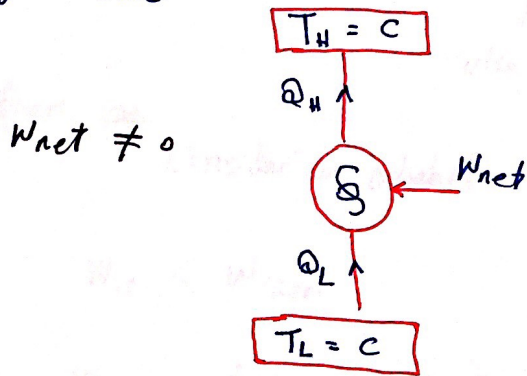
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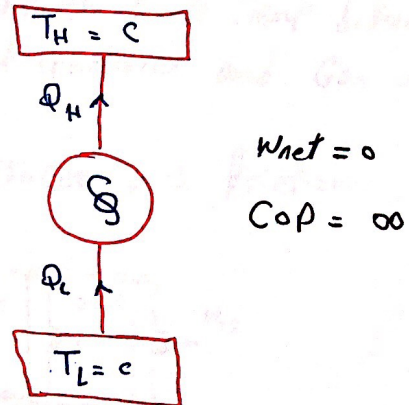
↪ Clausius statement ⇒

it is impossible to construct device and performs no effect other than transform of heat from lower Temp. body to the higher one



Ref. or H.P
satisfy clausius
statement

Vs

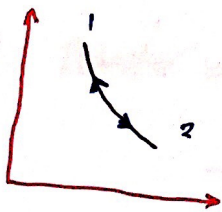


Ref. or H.P
doesn't satisfy clausius
statement

Reversible & irreversible Process ⇒

The reversible process is ⇒ A process that can be reversed without leaving any effect on the surrounding.

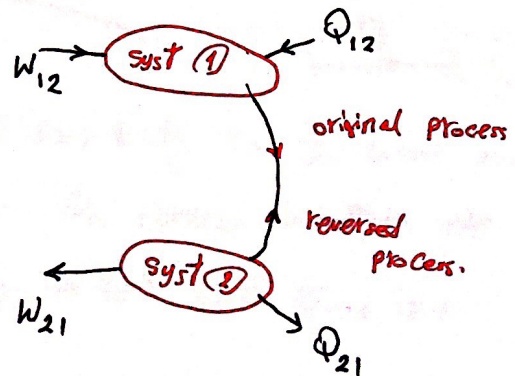
This mean that both the system and the surrounding must return to ^{their} initial state, and this



possible only

$$Q_{net} = Q_{12} + Q_{21} = 0$$

$$W_{net} = W_{12} + W_{21} = 0$$



Q2 The process that is not reversible is called (irreversible)

[9]

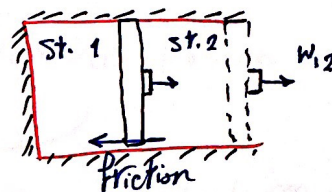
* **irreversibilities** \Rightarrow They are the factors that cause the process to be irreversible. They are: (Friction - Fast expansion & Compression - heat transfer through finite Temp different - also the chemical reactions and Gas mixing)

* **Friction** \Rightarrow

Consider an adiabatic expansion process with friction

$$W_{12} < W_{ideal}$$

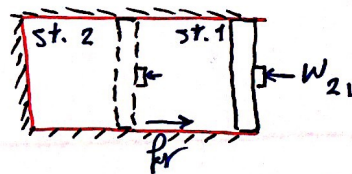
$$W_{12} = + (W_{ideal} - W_{fr})$$



if the process is reversed by adiabatic compression process.

$$W_{21} > W_{ideal}$$

$$W_{21} = - (W_{ideal} + W_{fr})$$



$$W_{net} = W_{12} + W_{21} = -2 W_{fr} \neq 0$$

So, The process with friction isn't reversible process

* **Heat transfer** \Rightarrow

Heat is transferred from high Temp body to the lower one.

To perform the process in the reverse direction we

need Ref or H. pump ($T_L \rightarrow T_H$) with $W_{net} \neq 0$



So, the process is irreversible process.



≠ Degree of irreversibility \Rightarrow

The higher Temp. different ($\Delta T = T_H - T_L$), the higher the degree of irreversibility

The lower Temp. different, the lower the degree of irreversibility
if we assumed that the heat transfer is due to differential Temperature difference dT

So,

What require to reverse the process can be considered reversible process.



The Reversible process \Rightarrow is Theoretical process, ideal process, can not performed practical so all the actual process are irreversible process.

Reversible process

Internally Reversible proc.

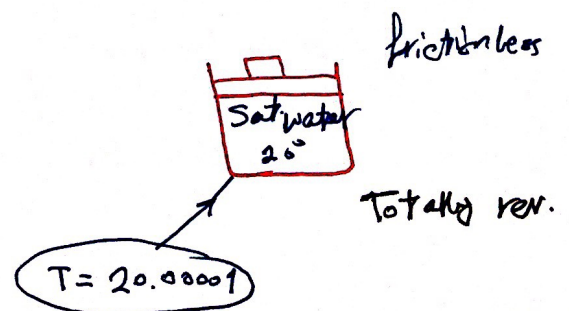
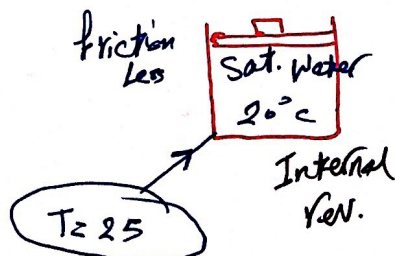
Externally Reversible

Totally Reversible.

Internally \Rightarrow it is the process that has no irreversibilities within the boundary of the system (process without friction)

Externally \Rightarrow the process that has no irreversibilities outside the boundary of the system. heat transfer is due to differential temp. different

Totally \Rightarrow the process that has no irreversibilities within or outside the boundary of the system



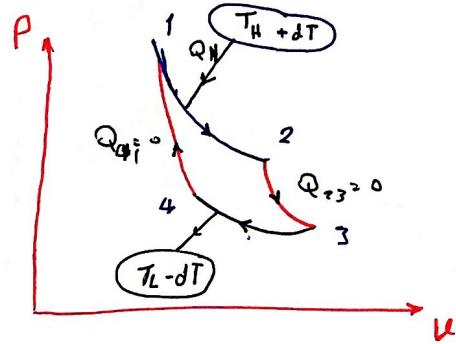


CarNot cycle \Rightarrow

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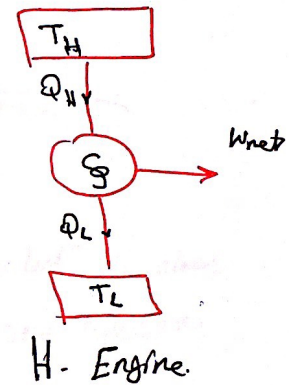
it consist of 4 reversible process.

- 1- Rev. isothermal expansion process (1-2)
 \Rightarrow heat addition process
- 2- Rev. adiabatic expansion process (2-3)
- 3- Rev isothermal compression process (3-4)
 \Rightarrow heat rejection process
- 4- Rev adiabatic compression process (4-1)



$$W_{net} = (W_{12} + W_{23}) - (W_{34} + W_{41}) = \text{area under curve}$$

\downarrow \downarrow
 W_{exp} W_{com}



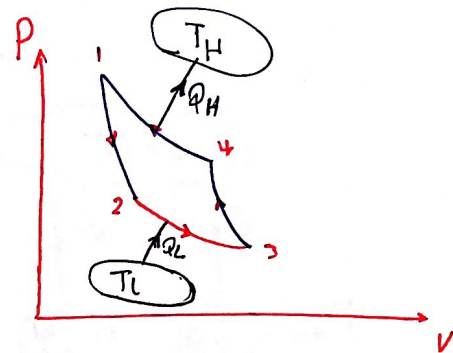
As shown the Carnot cycle represent heat engine. it is called "Carnot heat engine"
"rev heat engine"



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Reversed Carnot cycle \Rightarrow

- 1- Rev. adiabatic expansion process (1-2)
- 2- Rev. Isothermal expansion process (2-3)
 \Rightarrow heat addition process
- 3- Rev. adiabatic compression process (3-4)
- 4- Rev. Isothermal compression process (4-1)
 \Rightarrow heat rejection process



$$W_{net} = (W_{12} + W_{23}) + (W_{34} + W_{41}) = \text{area enclosed by curve } 12341 = -ve$$

\downarrow \downarrow
 $W_{exp.}$ $W_{com.}$

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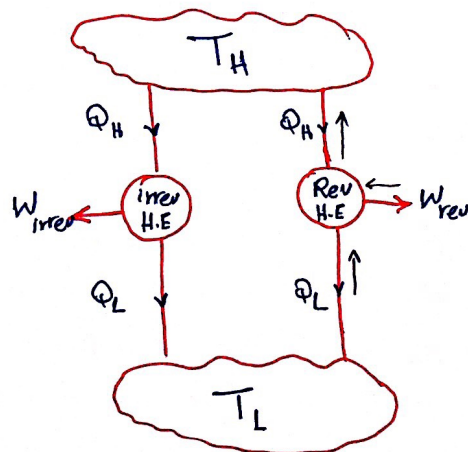
Carnot Principles \Rightarrow

- 1- The efficiency of an irreversible heat engine "actual" is always less than the efficiency of the reversible one operating between the same two reservoirs.

$$W_{irrev} < W_{rev}$$

$$Q_{L, irrev} > Q_{L, rev}$$

$$\eta_{th, irrev} < \eta_{th, rev}$$





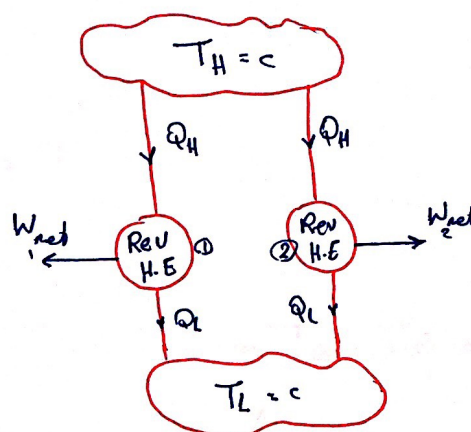
2. The efficiency of all reversible H.E operating bet. the same two reservoirs are the same.

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$$W_1 = W_2$$

$$Q_{L1} = Q_{L2}$$

$$\eta_{th,rev}^{(1)} = \eta_{th,rev}^{(2)}$$



The Thermodynamic Temperature scale \Rightarrow

From 2nd Carnot principle:-

$$\eta_{th,rev} = \text{const for the same } T_H, T_L$$

So, $\eta_{th,rev} = g(T_H, T_L)$
function of

$$\eta_{th,rev} = 1 - \left(\frac{Q_L}{Q_H} \right)$$

$$1 - \left(\frac{Q_L}{Q_H} \right)_{rev} = g(T_H, T_L)$$

$$\frac{Q_H}{Q_L} = \frac{1}{1 - g(T_H, T_L)}$$

We can say that

$$\left(\frac{Q_H}{Q_L} \right) = f(T_H, T_L)$$



Can prove that $\oint (T_H, T_L) = \frac{\phi(T_H)}{\phi(T_L)}$

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So,

$$\left(\frac{Q_H}{Q_L} \right)_{\text{rev}} = \frac{\phi(T_H)}{\phi(T_L)}$$

Kelvin proposed that $\phi(T) = T$

$$\left(\frac{Q_H}{Q_L} \right)_{\text{rev}} = \frac{T_{H(k)}}{T_{L(k)}}$$

$$\Rightarrow T_{(k)} = T_{(c)} + 273.15$$

Thermal efficiency of Rev H.E "Carnot" \Rightarrow

$$\eta_{\text{rev or irrev}} = 1 - \frac{Q_L}{Q_H}$$

$$\eta_{\text{rev}} = 1 - \frac{T_{L(k)}}{T_{H(k)}}$$

So, for any H.E

$$\eta_{\text{H.E}} \begin{cases} < \eta_{\text{rev}} \Rightarrow \text{Actual irrev. H.E} \\ = \eta_{\text{rev}} \Rightarrow \text{Rev H.E} \\ > \eta_{\text{rev}} \Rightarrow \text{impossible H.E} \end{cases}$$



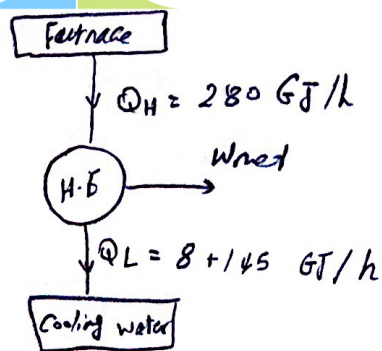
1) W_{out} , η_{th}

Sol

$$\dot{Q}_H - \dot{Q}_L = W_{out}$$

$$W_{out} = 280 - 153 = 127 \text{ GJ/h} \xrightarrow{60 \times 60} \frac{10^9}{\text{s}} \text{ J/s (Watt)}$$

$$\eta_{th} = \frac{W_{out}}{\dot{Q}_H} = \frac{127}{280} = 0.45 \%$$



[2]

$$W_{out} = 90 \text{ kW}$$

$$\eta_{th} = 0.28$$

1 kg مقدار الحرج الناتج من حرج ٢.٤١٥٥ $H.V = 44000 \text{ kJ/kg}$

\dot{m}_f ?

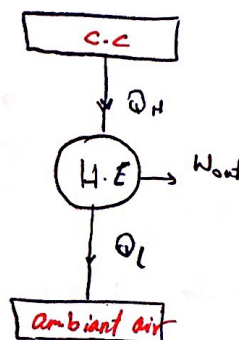
Sol

$$\dot{Q}_H = \dot{m}_f \times H.V_f$$

$$\text{kg/s} \cdot \text{kJ/kg} \Rightarrow \text{kJ/s Watt}$$

$$\eta_{th} = \frac{W_{out}}{\dot{Q}_H}, \quad \dot{Q}_H = \frac{90}{0.28} = 3214 \text{ kW}$$

$$\dot{m}_f = \frac{\dot{Q}_H}{H.V_f} = 7.3 \times 10^{-3} \text{ kg/s} = 26.2 \text{ kg/h}$$



[4]

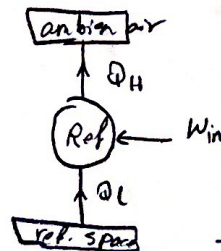
$$\dot{Q}_L = 8000 \text{ kJ/h} \times \frac{1}{60 \times 60} =$$

$$W_{in} = 1 \text{ kW}$$

$$\dot{Q}_H = ?, \quad \text{COP} = ?$$

$$\text{COP}_R = \frac{\dot{Q}_L}{W_{in}} = \frac{8000}{60 \times 60 \times 1} = 2.2$$

$$\dot{Q}_H = W_{in} + \dot{Q}_L = 3.2 \text{ kW}$$



$$\eta < 1$$

$$\text{COP} > 1$$

$$TR \Rightarrow 3.5 \text{ kW}$$



[7]

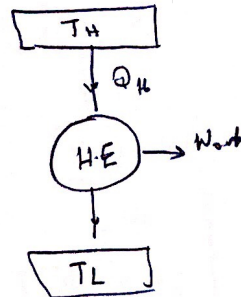
$$\eta = 0.55$$

$$T_L = 15 + 273$$

$$Q_L = 800 \text{ kJ/min}$$

$$W_{out} = ?$$

$$T_H = ?$$



$$\eta_{H.E. Rev} = \frac{T_H - T_L}{T_H}, \quad T_H = 640 \text{ K}$$

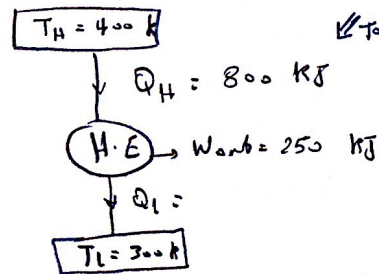
$$\frac{Q_H}{Q_L} = \frac{T_H}{T_L} \Rightarrow Q_H = 1777 \text{ kJ/min}$$

$$W_{out} = Q_H - Q_L = \text{kJ/min}$$

[8]

$$\eta_{H.E. rev} = \frac{T_H - T_L}{T_H} = 0.25$$

$$\eta_{H.E. inv} = \frac{W_{out}}{Q_H} = \frac{250}{800} = 0.31$$



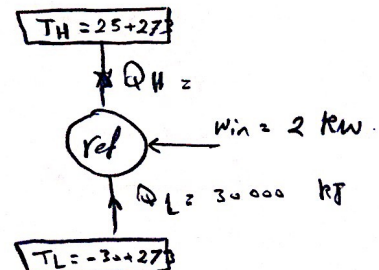
duration time
total amount.

$$\eta_{H.E. inv} > \eta_{H.E. rev} \Rightarrow \text{impossible cycle.}$$

[12]

$$\text{time} = 20 \text{ min}$$

$$(COP)_{ref}^{rev} = \frac{T_L}{T_H - T_L} = \frac{243}{298 - 243} = 4.4$$



$$(COP)_{ref}^{act} = \frac{Q_L}{W_{in}} = \frac{30,000}{20 \times 60} = 12.5$$

$$\therefore (COP)_{inv} > (COP)_{rev}$$

impossible cycle.

$$W_{in} = 2 \text{ kW} \times 20 + 60 = \text{kJ}$$



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Ref $Q_H = ?$

$Q_L + Q_H' = ?$

rev. ref

$$COP_{rev} = \frac{T_L}{T_H - T_L} = 6.14$$

$$COP_{rev} = \frac{Q_L}{W_{in}} \Rightarrow W_{in} = 65.15 \text{ kJ/min}$$

$$Q_H' = W_{in} + Q_L' = 465.15 \text{ kJ/min}$$

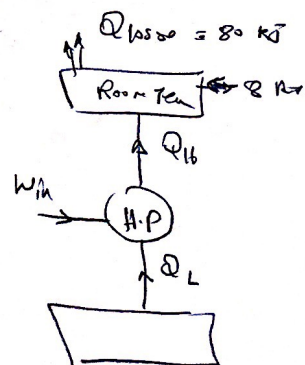
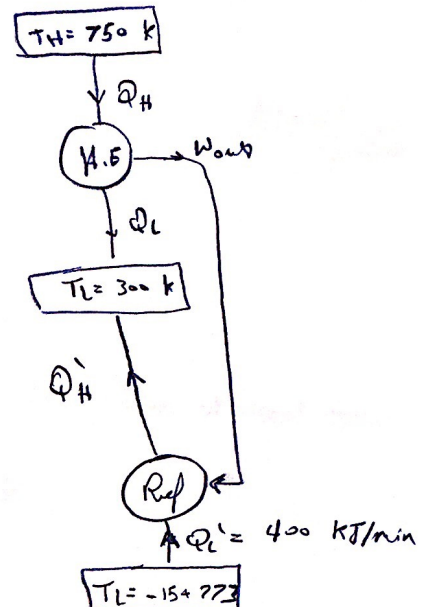
H.E rev

$$W_{in, Ref} = W_{out, H.E}$$

$$\eta_{H.E, rev} = \frac{T_H - T_L}{T_H} = 0.6 = \frac{W_{out}}{Q_H}$$

$$Q_H = 108.6 \text{ kJ/min}, Q_L = 43.4 \text{ kJ/min}$$

$$Q_H = Q_{L, loss} - Q_g = 72 \text{ kJ}$$





Thermo. B

28/2

Ch. 2

"Entropy"

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①

The Clausius inequality \Rightarrow

The cyclic integration of $\oint \frac{\delta Q}{T} \leq 0$ Less than or equal zero.

This inequality is valid for all cycle.

For irrev. cycle $\Rightarrow \oint \frac{\delta Q}{T} < 0$

For rev cycle $\Rightarrow \oint \frac{\delta Q}{T} = 0$

For impossible cycle $\Rightarrow \oint \frac{\delta Q}{T} > 0$

* For Heat Engine \Rightarrow

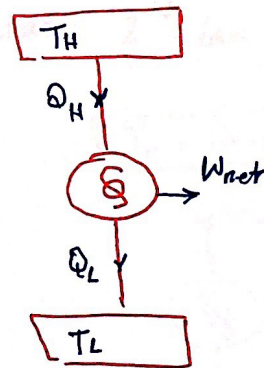
$$\oint \frac{\delta Q}{T} = \left[\frac{Q_H}{T_H} - \frac{Q_L}{T_L} \right]$$

if $\left[\frac{Q_H}{T_H} - \frac{Q_L}{T_L} \right] < 0$, actual irrev. H.E

if $\left[\frac{Q_H}{T_H} - \frac{Q_L}{T_L} \right] = 0$, rev. H.E

$$\frac{Q_H}{T_H} = \frac{Q_L}{T_L}, \left(\frac{Q_H}{Q_L} \right)_{\text{rev}} = \frac{T_H}{T_L} \Rightarrow \text{Carnot principle}$$

if $\left[\frac{Q_H}{T_H} - \frac{Q_L}{T_L} \right] > 0$, impossible cycle





* For Refrigerator or Heat Pump \Rightarrow

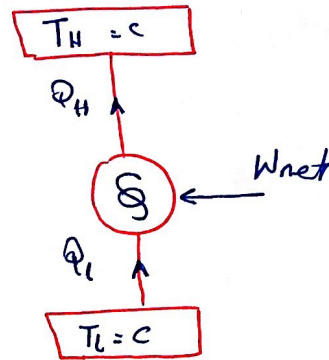
(2)

$$\oint \frac{\delta Q}{T} = \left[\frac{Q_L}{T_L} - \frac{Q_H}{T_H} \right]$$

if $\left[\frac{Q_L}{T_L} - \frac{Q_H}{T_H} \right] < 0$, actual irrev. Ref. H.P.

if $\left[\frac{Q_L}{T_L} - \frac{Q_H}{T_H} \right] = 0$, rev. Ref. H.P.

if $\left[\frac{Q_L}{T_L} - \frac{Q_H}{T_H} \right] > 0$, impossible cycle.



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\Rightarrow Equivalence between Carnot principle & Clausius inequality

For the shown H.E determine if it violates 2nd law on the bases of - Carnot & clausius

From 1st Law of Therm

$$Q_H = Q_L + W_{net}$$

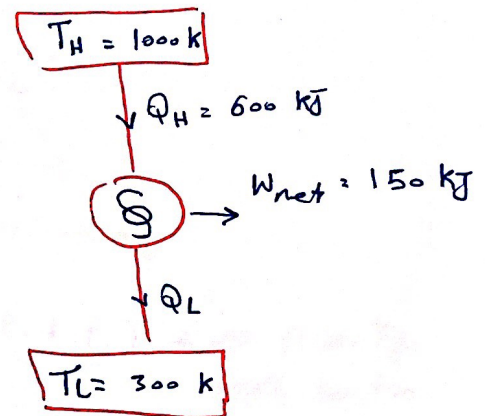
$$Q_L = 600 - 150 = 450 \text{ kJ}$$

* Carnot \Rightarrow

$$\eta_{th} = \frac{W_{net}}{Q_H} = \frac{150}{600} = 0.25$$

$$\eta_{th,rev} = 1 - \frac{T_L (K)}{T_H (K)} = 1 - \frac{300}{1000} = 0.7$$

$\therefore \eta_{th} < \eta_{th,rev} \Rightarrow$ H.E is irrev it doesn't violates.



* Clausius inequality \Rightarrow

$$\oint \frac{\delta Q}{T} = \frac{Q_H}{T_H} - \frac{Q_L}{T_L} = 0.6 - 1.5 = -0.9 < 0, \text{ actual irrev. H.E}$$



For the shown Heat Pump

From 1st law

$$Q_L = 450 \text{ kJ}$$

* Carnot \Rightarrow

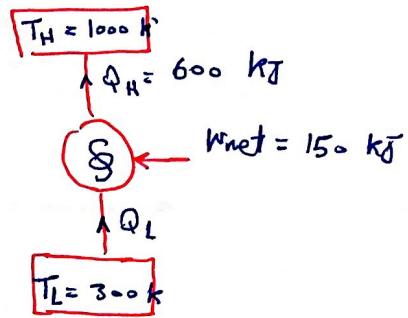
$$\text{COP}_{\text{H.P.}} = \frac{Q_H}{W_{\text{net}}} = 4$$

$$\text{COP}_{\text{H.P. rev}} = \frac{1}{1 - \frac{T_L}{T_H}} = 1.4$$

* Clausius \Rightarrow

$$\oint \frac{\delta Q}{T} = \frac{Q_L}{T_L} - \frac{Q_H}{T_H} = 1.5 - 0.6 = 0.9 > 0, \text{ impossible H.P.}$$

$\text{COP}_{\text{H.P.}} > \text{COP}_{\text{rev}} \Rightarrow$ impossible cycle
H.P. violates 2nd law



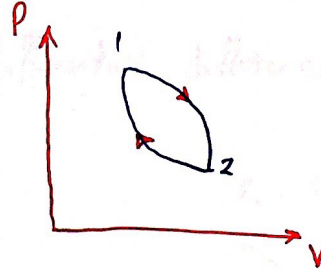
Entropy \Rightarrow

The clausius inequality represent the basic for the definition of new property called entropy.

Consider a cycle as shown

$$\oint dv = \int_1^2 dv + \int_2^1 dv = (v_2 - v_1) + (v_1 - v_2) = 0$$

$$\oint du = \int_1^2 du + \int_2^1 du = (u_2 - u_1) + (u_1 - u_2) = 0$$



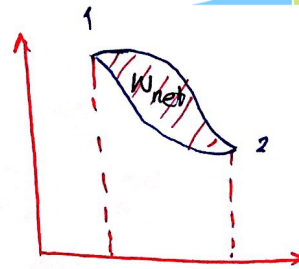
$u, v, h, p, T \Rightarrow$ are property
state function.
Point function.

So \Rightarrow The cyclic integration of any property equal Zero.



$$\oint \delta w = \int_1^2 \delta w + \int_2^1 \delta w$$

$$= w_{1,2} + w_{2,1} = w_{net} \neq 0$$



$$\oint \delta Q = \int_1^2 \delta Q + \int_2^1 \delta Q$$

$$= Q_{1,2} + Q_{2,1} = Q_{net} \neq 0$$

Because Q, w are not properties, they are (Path function)

So, we conclude that..

⇒ if the cyclic integration of a quantity equal zero

→ This quantity is a property of a system

* For rev. cycle ⇒

$$\oint \left(\frac{\delta Q}{T} \right)_{rev} = 0$$

This mean that $\left(\frac{\delta Q}{T} \right)_{rev}$ represent differential difference of This

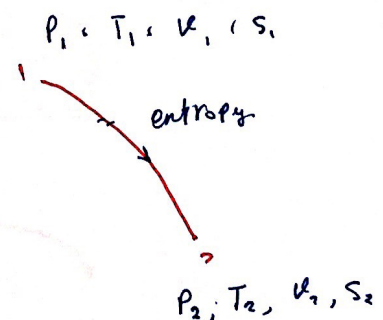
new property "S" called entropy

So $ds = \left(\frac{\delta Q}{T} \right)_{rev}$

$$\Delta s = \int_1^2 ds = \int_1^2 \left(\frac{\delta Q}{T} \right)_{inter rev} \quad \text{kJ / K}^{\circ}$$

Per unit mass

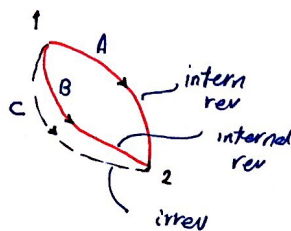
$$\Delta s = \int_1^2 \left(\frac{\delta Q}{T} \right)_{inter rev} \quad \text{kJ / kg. K}^{\circ}$$





if we have

5



$$\Delta S)_A = \int_1^2 \left(\frac{\delta Q}{T} \right)_{\text{intern rev}} = S_2 - S_1$$

$$\Delta S)_B = \int_1^2 \left(\frac{\delta Q}{T} \right)_{\text{internal rev}} = S_2 - S_1$$

$$\Delta S)_C \neq \int_1^2 \left(\frac{\delta Q}{T} \right)_{\text{irrev}} = \Rightarrow \Delta S)_C = \Delta S)_B = \Delta S)_A$$

Special process \Rightarrow

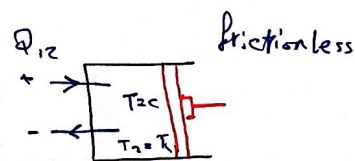
internal rev isothermal process ($T=c$)

$$\Delta S = \int \left(\frac{\delta Q}{T} \right)_{\text{int rev}}, T=c$$

$$\Delta S = \frac{1}{T} \int \delta Q)_{\text{int. rev}} = \pm \frac{Q_{12}}{T}$$

$$S_2 - S_1 = \pm \frac{Q_{12}}{T}$$

+ heat addition \rightarrow increase S^{in} entropy





6

Thermo

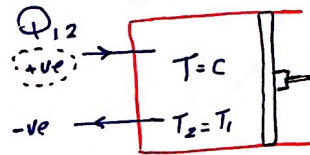
الترموديناميكس

2/3/2016

* Int. rev. isothermal (heat addition or heat rejection) process

$$\Delta S = \int_1^2 \frac{\delta Q}{T} \Big|_{\text{int. rev}}$$

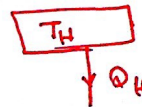
$$\Delta S = \frac{1}{T} \int_1^2 \delta Q \Big|_{\text{int. rev}} = \pm \frac{Q_{12}}{T}$$



→ This equ. can be used to determination of change of entropy

1. Source ⇒

$$\Delta S_{\text{source}} = \frac{-Q_H}{T}$$



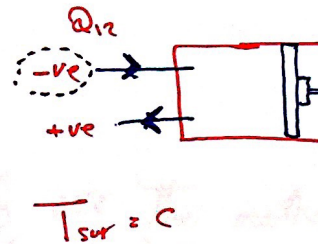
2. Sink ⇒

$$\Delta S_{\text{sink}} = \frac{+Q_L}{T}$$



#3. Surrounding ⇒

$$\Delta S_{\text{sur}} = \pm \frac{Q_{12}}{T_{\text{sur}}} \text{ kJ/K}$$



Ahmed

Kasem



* The increase of entropy principle \Rightarrow

(7)

Consider a cycle as shown

$$\oint \frac{\delta Q}{T} \leq 0$$

$$\int_1^2 \frac{\delta Q}{T} + \int_2^1 \frac{\delta Q}{T} \Big|_{\text{int. rev}} \leq 0$$

$$= \int_1^2 \frac{\delta Q}{T} + (s_2 - s_1) \leq 0$$

$$s_2 - s_1 \geq \int_1^2 \frac{\delta Q}{T}$$

$$\Delta s \geq \int_1^2 \frac{\delta Q}{T}$$

\Rightarrow for int. rev

$$\Delta s = \int_1^2 \frac{\delta Q}{T}$$

\Rightarrow for irrev.

$$\Delta s > \int_1^2 \frac{\delta Q}{T}$$

* for any process \Rightarrow

$$\Delta s \geq \int \frac{\delta Q}{T}$$

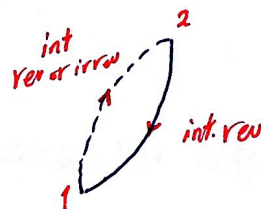
or differential form

$$ds \geq \frac{\delta Q}{T}$$

This eqn. can be written as

$$\Delta s = \int_1^2 \frac{\delta Q}{T} + S_{\text{gen}}$$

They are the mathematical statement of the 2nd law.





$$\Delta S = \int_1^2 \frac{\delta Q}{T} + S_{gen}$$

⑧

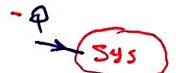
$S_{gen} = 0 \Rightarrow$ for int. rev process

$S_{gen} > 0 \Rightarrow$ for irrev. process.

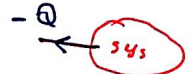
It is the entropy generation due to irreversibilities only "friction"

$\int_1^2 \frac{\delta Q}{T} \Rightarrow$ entropy transfer with heat, it has the same sign
* Convention as heat

① $\int_1^2 \frac{\delta Q}{T} = +ve$, if the heat add to the system (+) it causes increase of entropy.



② $\int_1^2 \frac{\delta Q}{T} = -ve$, if the heat reject from the system (-) it decrease of entropy.



③ $\int_1^2 \frac{\delta Q}{T} = 0$, for adiabatic process. $Q=0$

For isolated system \Rightarrow

$$\Delta S_{isol} = \int \frac{\delta Q}{T} + S_{gen} \geq 0$$

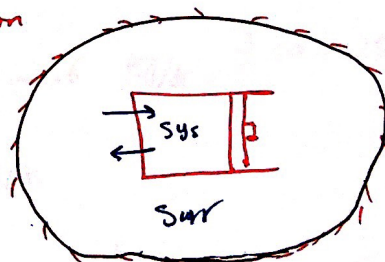
$$\Delta S_{isol} = S_{gen} \geq 0 \Leftarrow$$

if we take the isolation system as shown

$$\Delta S_{tot} = \Delta S_{sys} + \Delta S_{surr} = S_{gen}$$

$S_{gen} = 0$, the process is rev

$S_{gen} > 0$, ~ ~ ~ irrev



isolated
syst



All The actual processes are irrev. processes.

So $\Delta s)_{tot}$ for any actual process must be ≥ 0

So, for any process

$$\Delta s)_{tot} \begin{cases} > 0 \Rightarrow \text{actual irrev. process.} \\ = 0 \Rightarrow \text{Rev. process.} \\ < 0 \Rightarrow \text{Impossible process.} \end{cases}$$

Determination of $\Delta s)_{tot} \Rightarrow$

① for closed system \rightarrow

$$\Delta s)_{tot} = \Delta s)_{sys} + \Delta s)_{surr}$$

where

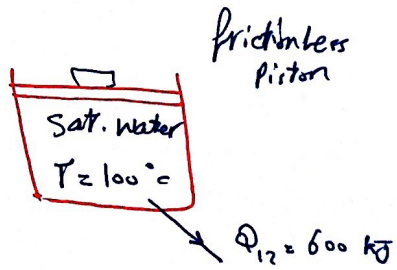
$$\Delta s)_{sys} = \int \frac{\delta Q}{T} + S_g$$

$$\Delta s)_{surr} = \frac{\pm Q}{T}$$

Ex

Const pressure cooling process of sat. water

Det. change of entropy - $\Delta s)_{w}$ ($\Delta s)_{surr}$) Type of process.



$$\Delta s)_{w} = \int_1^2 \frac{\delta Q}{T} + S_g = \frac{1}{T_w} \int \delta Q = \frac{Q_{12}}{T_w} = \frac{-600}{100 + 273} = -1.6 \text{ kJ/K}$$

$T_{surr} = 25^\circ$

$$\Delta s)_{surr} = \frac{+Q_{12}}{T_{surr}} = \frac{600}{25 + 273} = 2.01 \text{ kJ/K}$$

$$\Delta s)_{tot} = -1.6 + 2.01 = 0.4 > 0 \text{ actual irrev process.}$$



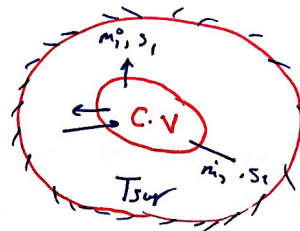
② for open system →

(10)

$$\Delta s)_{\text{Tot}} = \Delta s)_{\text{C.V}} + \Delta s)_{\text{sur}}$$

in rate form

$$\frac{ds)_{\text{tot}}}{dt} = \frac{ds)_{\text{C.V}}}{dt} + \frac{ds)_{\text{sur}}}{dt}$$



Isolated System

for steady state condition

$$\frac{d}{dt} = 0$$

$$\frac{dm_{\text{C.V}}}{dt} = 0$$

$$\Rightarrow \dot{m}_{\text{in}} = \dot{m}_{\text{out}}$$

also

$$\frac{ds_{\text{C.V}}}{dt} = 0$$

$$\frac{dE_{\text{C.V}}}{dt} = 0$$

$$E_{\text{in}} = E_{\text{out}}$$

$$\begin{aligned} \dot{S}_{\text{tot}} &= \frac{ds)_{\text{tot}}}{dt} = \frac{ds_{\text{C.V}}}{dt} + \frac{ds)_{\text{sur}}}{dt} \\ &= \dot{m}s_2 - \dot{m}s_1 + \frac{\dot{Q}}{T_{\text{sur}}} \end{aligned}$$

$$\boxed{= \dot{m}(s_2 - s_1) + \frac{\dot{Q}}{T_{\text{sur}}}}$$

Per unit mass

$$\Delta s_{\text{tot}} = (s_2 - s_1) + \frac{q}{T_{\text{sur}}}$$

So,

$$\Delta s_{\text{tot}} \begin{cases} > 0 \Rightarrow \text{actual irrev process} \\ = 0 \Rightarrow \text{Rev. pro} \\ < 0 \Rightarrow \text{impossible pro} \end{cases}$$



Thermo

الأحرار
6/3/2016

(11)

Third law of Thermodynamics \Rightarrow

The entropy of a pure crystalline substance at absolute zero temp.
(substance with perfect order) is zero.

\Rightarrow Entropy: can be defined as \Rightarrow the ^{measure} major of degree of disorder.

Diagrams involving Entropy \Rightarrow

it's useful to draw a process in diagrams one of their coordinates is entropy. $T-s$ and $h-s$

$\Rightarrow T-s$ diagram \rightarrow

from 2nd law

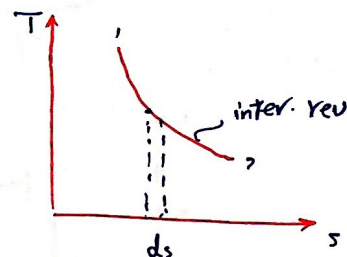
$$\Delta s = \int \left(\frac{\delta Q}{T} \right)_{\text{int. rev}}$$

$$ds = \frac{\delta Q}{T}_{\text{int. rev}}$$

$$\delta Q = T ds = \text{differential}$$

area element on $T-s$ diag.

$$\int \delta Q_{\text{int. rev}} = \int T \cdot ds = \text{area under process curve}$$



So, we conclude that

The area under process curve on $T-s$ diagram represent the heat transfer for int. rev process



11 Int. rev isothermal process \Rightarrow

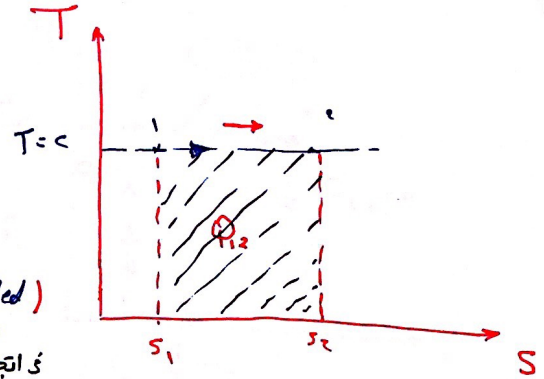
(12)

$$Q_{12}^{\text{int.rev}} = \int_1^2 T \cdot ds$$

$$= T \int_1^2 ds$$

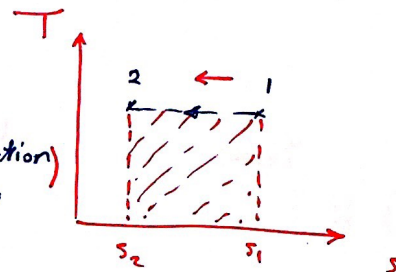
$$Q_{12}^{\text{int.rev}} = T(s_2 - s_1) = +ve \quad (\text{heat added})$$

process
في اتجاه زيادة S



$$Q_{12}^{\text{int.rev}} = T(s_2 - s_1) = -ve$$

(heat rejection)
process



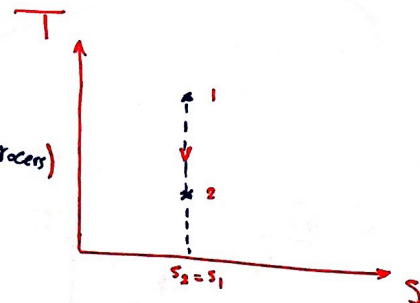
2. Int. rev adiabatic process \Rightarrow

$$\Delta s = \int \frac{\delta Q}{T} + S_g$$

$$\Delta s = 0$$

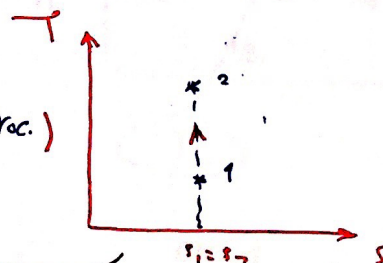
$$s_2 - s_1 = 0 \quad \therefore s_2 = s_1$$

(expansion process)



Const entropy
process, (isentropic process)

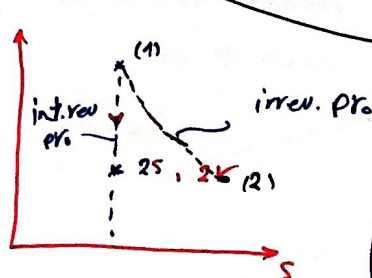
(compression proc.)



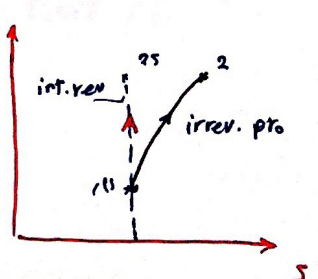
3. Adiabatic process

Friction

$$\Delta s = \int \frac{\delta Q}{T} + S_g > 0$$



$$\Delta s = \int \frac{\delta Q}{T} + S_g > 0$$





*b Show The carnot cycle on T-s diagram \Rightarrow

(13)

$$Q_{net} = Q_{12} + Q_{23} + Q_{34} + Q_{41}$$

$$= Q_H + Q_L$$

Q_H = area under curve process (1-2)

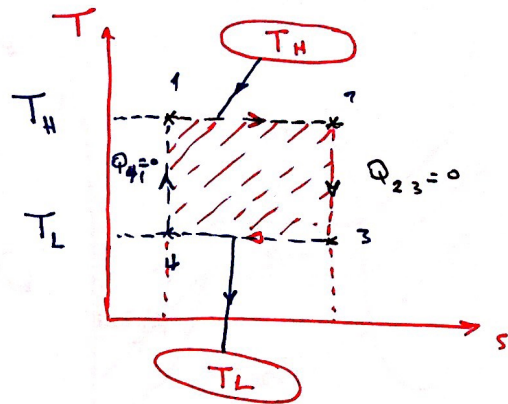
$$T_H (s_2 - s_1) = +ve$$

Q_L = area under curve process (3-4)

$$T_L (s_4 - s_3) = -ve$$

$$Q_{net} = (T_H - T_L) \Delta s =$$

Q_{net} = area enclosed by the curve.



Carnot (H.E)

Reversed carnot cycle on T-s diag \Rightarrow

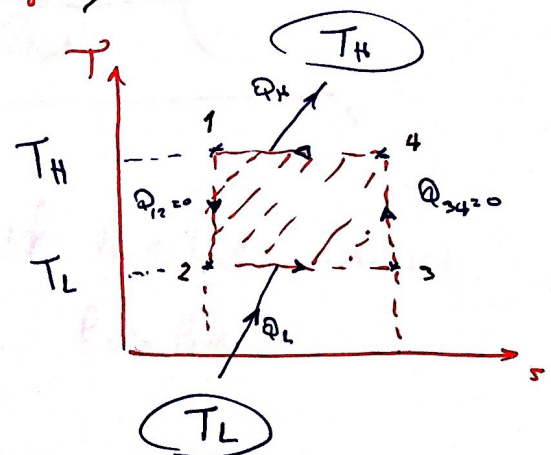
$$Q_{net} = Q_{12} + Q_{23} + Q_{34} + Q_{41}$$

$$= Q_{23} + Q_{41}$$

$$= T_L (s_3 - s_2) + T_H (s_1 - s_4)$$

+ve -ve

$$= (T_L - T_H) \Delta s = \text{area enclosed by the curve.}$$



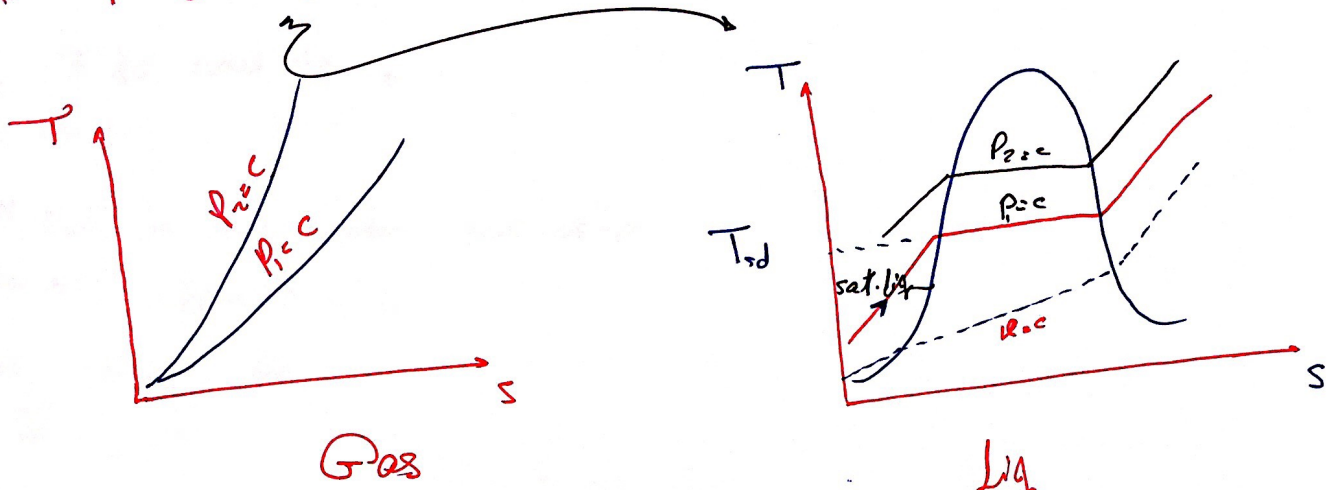
Carnot (Ref), (H.P)

Ahmed

Kasem



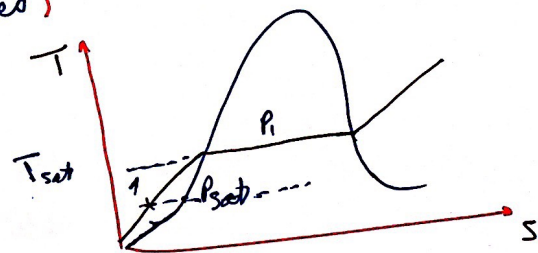
91 * T-S diagram for pure substance \Rightarrow



Gas
 $P_2 > P_1$
 $u_2 < u_1$

Liq
 $P_2 > P_1$
 $u_2 < u_1$

* Compressed Liquid (sub cooled)



for no available data
we will consider state (1)
compressed liquid =
state (f) @ T_1 of liquid

if $T_1 < T_{sat}$, subcooled
 $P_1 > P_{sat}$

$$\left. \begin{array}{l} u_1 = u_f \\ @ T_1 \end{array} \right\} \begin{array}{l} u_1 = u_f \\ @ T_1 \\ h_1 = h_f \\ @ T_1 \end{array}$$



Thermo

9/3

الحرارة

The Tds relations \Rightarrow

1st law for closed system under int. rev process, differential form:..

$$(\delta Q)_{\text{int rev}} - (\delta W)_{\text{int rev}} = du$$

$$\Rightarrow (\delta W)_{\text{int rev}} = Pdv$$

$$\delta Q)_{\text{int rev}} - Pdv = du$$

for 2nd law

$$\delta Q)_{\text{int rev}} = T \cdot ds$$

$$T \cdot ds = du + Pdv$$

for unit mass

$$T \cdot ds = du + Pdv \Rightarrow ①$$

$$h = u + Pv$$

تفاضل

$$dh = \underbrace{du + Pdv}_{T \cdot ds} + vdp$$

So

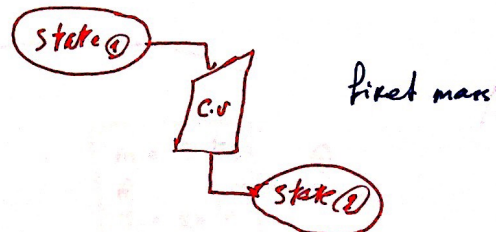
$$T \cdot ds = dh - vdp \Rightarrow ②$$

function of
initial & state
final state

The above eqn. are applicable for a closed system when its state is changed from state 1 to state 2

It's applicable for open system (Control volume).

When the inlet mass is changed from state ① to ②



Change of entropy for pure substance \Rightarrow

$$\Delta s = \int_1^2 ds = \int_1^2 \frac{du}{T} + \int_1^2 \frac{P du}{T} \rightarrow \text{①}$$

$$\Delta s = \int_1^2 ds = \int_1^2 \frac{dh}{T} - \int_1^2 v dp \rightarrow \text{②}$$

\rightarrow To perform these integration the relation of $(u-T)$, $(h-T)$, $(P-v-T)$ must be known.

\rightarrow For pure substances these relation are very complicated
the integrations were performed and the results given in table and charts.

Isentropic process for pure substances \Rightarrow

$$\Delta S = m(s_2 - s_1)$$

↳ mass of system.

Isentropic for the process must be reversible adiabatic process

$$s_2 = s_1$$

$$\Delta s = 0 \quad \rightarrow \text{rev. adiabatic process.}$$

many devices actually operate under adiabatic process (Compressor)

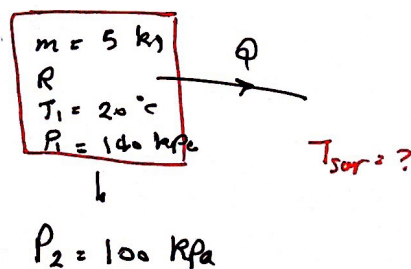
So, the Isentropic process (int. rev adiabatic) represent the ideal process for these devices

Ex Rigid tank

Ref. ① change of entropy Δs .

② Q , amount of heat rejected.

③ Type of process





state ①

$$P_1 = 140 \text{ kPa}$$

$$T_1 = 20^\circ \text{C}$$

$$T_{\text{sat}} = -21.91^\circ \text{C}$$



$T_1 > T_{\text{sat}}$
superheated

from superheated tables..

by $P_1 = 140 \text{ kPa}$

$$T_1 = 20^\circ \text{C}$$

$$u_1 = 0.387 \text{ m}^3/\text{kg}$$

$$s_1 = 0.8035 \text{ kJ/kg}\cdot\text{K}$$

$$u_1 = 103.87 \text{ kJ/kg}$$

state ②

$$P_2 = 100 \text{ kPa}$$

$$u_2 = u_1$$

$$u_2 = 0.1397$$

from sat tables

$$u_f \text{ at } P_2 =$$

$$u_g \text{ at } P_2 =$$

sat. mixture

$$\Delta s = s_2 - s_1 = 0.63 - 0.80 = -ve$$

$$\Delta S = m \Delta s$$

$$u_2 = u_f + x_2(u_g - u_f)$$

$$x_2 =$$

$$s_2 = s_f + x_2(s_g - s_f)$$

$$s_2 = 0.6307 \text{ kJ/kg}\cdot\text{K}$$

$$u_2 =$$

$$Q_{12} - W_{12} = m(u_2 - u_1) = -ve$$

$$\Delta S_{\text{sur}} = \frac{Q_{12}}{T_{\text{sur}}} = \frac{+ve}{T_{\text{sur}}} =$$

$$\Delta S_{\text{tot}} = \Delta S_{\text{sys}} + \Delta S_{\text{sur}} > 0$$

13/3/19

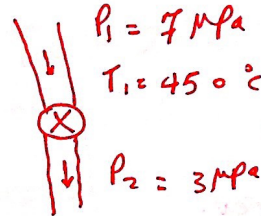
$h = \text{const}$

Ex. 2

Steam is Throttled from

$P_1 = 7 \text{ MPa}$ to

$P_2 = 3 \text{ MPa}$



Req.

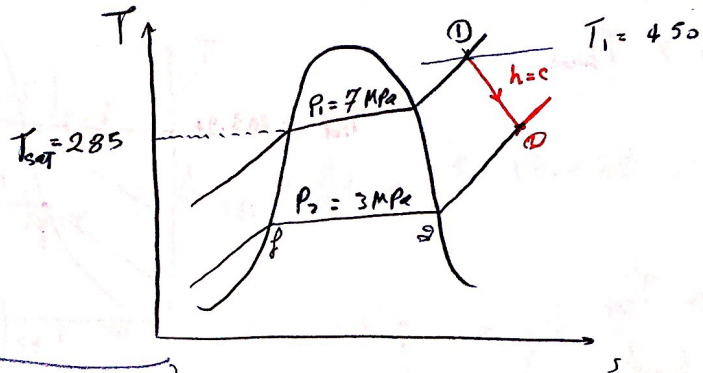
1- Δs

2- Type of process

R.

$P_1 = 7 \text{ MPa}$

$T_1 = 450^\circ \text{C}$



T_{sat} at $P_1 = 7 \text{ MPa}$

$T_{\text{sat}} = 285$

$T_1 > T_{\text{sat}}$
super heat

Table: By $P_1 = 7 \text{ MPa}$
 $T_1 = 450^\circ \text{C}$

$s_1 = 6.6327 \text{ kJ/kg}\cdot\text{K}$

$h_1 = 3287.1 \text{ kJ/kg}$

$h_1 = h_2$

$$\Delta s_{\text{tot}} = (s_2 - s_1) + \frac{q}{T_{\text{sur}}} = +ve$$

actual irrev.

very rapid process \Rightarrow adiabatic process

$h_{\text{transfer}} = 0$

state 2

$P_2 = 3 \text{ MPa}$

$h_2 = 3287.1 \text{ kJ/kg}$

from sat. tables

by $P_2 = 3 \text{ MPa}$

h_f

$h_g = 2803.2$

$h_2 > h_g \Rightarrow$ superheated table

from P_2, h_2

$P_2 = 3 \text{ MPa}$

$h_2 = 3287.1$

$s_2 =$	450	3231.7	6.9235
$T_2 =$	450	3287.1	s_2
	450	3344.9	7.0856

$s_2 = 6.9919 \text{ kJ/kg}\cdot\text{K}$

$T_2 = -$



3. steam turbine

$$P_1 = 5 \text{ MPa}$$

$$T_1 = 450^\circ\text{C}$$

$$P_2 = 1.4 \text{ MPa}$$

consider is int. rev.

Req.

Wout & T_{finel}

$$T_{\text{sat}} = 263.94^\circ\text{C}$$

$$T_1 > T_{\text{sat}}$$

super heat

from super heated table

$$P_1 = 5 \text{ MPa}$$

$$T_1 = 450^\circ\text{C}$$

$$s_1 = 6.8210 \text{ kJ/kg}\cdot\text{K}$$

$$h_1 = 3317.2 \text{ kJ/kg}$$

rev. adiabatic \Rightarrow isentropic.

$$s_2 = s_1$$

state

$$P_2 = 1.4 \text{ MPa}$$

$$s_2 = 6.8210 \text{ kJ/kg}\cdot\text{K}$$

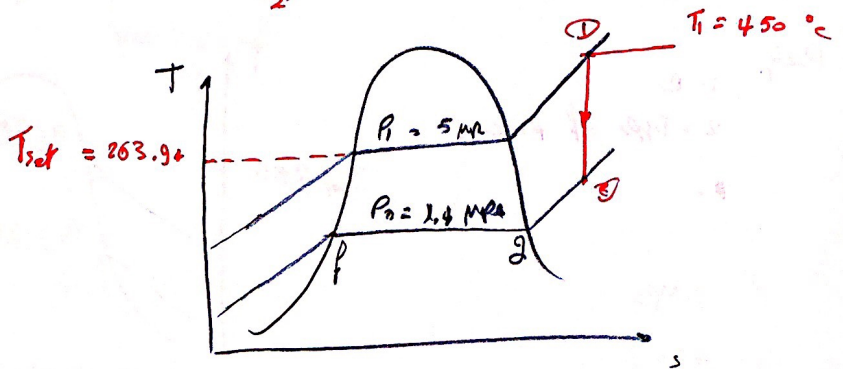
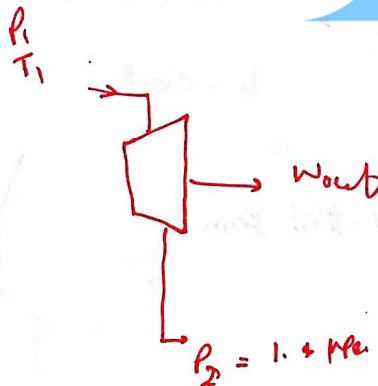
sat. table

$$\text{by } P_2 = 1.4 \text{ MPa}$$

$$s_f =$$

$$s_g = 6.4673 \text{ kJ/kg}\cdot\text{K}$$

$$s_2 > s_g \Rightarrow \text{super heated}$$



$$P_2 = 1.4 \text{ MPa}$$

h	s	T
2927.9	6.7488	250
h2	6.8210	T2
3040.9	6.9553	300

$$T_2 =$$

$$\frac{h_2 - 2927.9}{3040.9 - 2927.9} = \frac{6.8210 - 6.7488}{6.9553 - 6.7488}$$

$$h_2 = 2967.4 \text{ kJ/kg}$$

$$T_2 = 1$$

is la

$$h_1 - h_2 = w_{12}$$

$$h_1 - h_2 = w_{12} = 349.8 \text{ kJ/kg}$$

$$\text{Power} = \dot{m} \times W_{\text{out}} =$$

$$= \text{kW}$$

② R-134a

* closed system

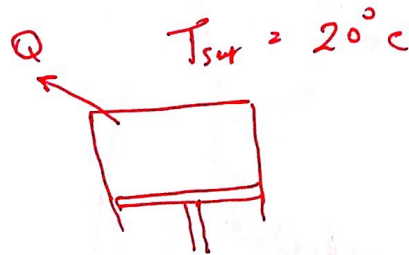
$$m = 5 \text{ kg}$$

$$P_1 = 0.8 \text{ MPa}$$

$$T_1 = 50^\circ \text{C}$$

$$\text{Const } P \Rightarrow P_1 = P_2$$

Liquid $T_2 = 24^\circ \text{C}$



$$\Delta s)_{\text{tot}} ?$$

$$\Delta s)_{\text{tot}} = \Delta s_{\text{sys}} + \Delta s_{\text{sur}}$$

$$\Delta s)_{\text{sys}} \Rightarrow$$

state ①

$$P_1 = 0.8 \text{ MPa}, T_1 = 50^\circ \text{C}$$

by using sat. Table R134a (A-12)

$$@ P_1 = 0.8 \text{ MPa} \rightarrow T_{\text{sat}} = 31.31^\circ \text{C}$$

$$T_1 > T_{\text{sat}}$$

by using superheat table (A-13)

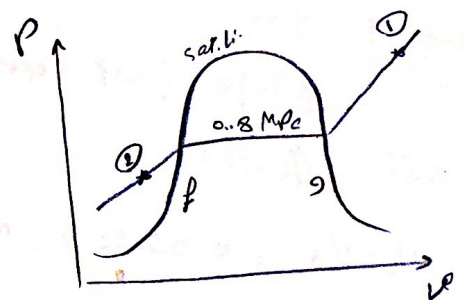
$$P_1 = 0.8 \text{ MPa}$$

$$T_1 = 50^\circ \text{C}$$

$$u_1 = 263.86 \text{ kJ/kg}$$

$$v_1 = 0.028547 \text{ m}^3/\text{kg}$$

$$s_1 = 0.9802 \text{ kJ/kg}\cdot\text{K}$$



state ②

$$P_2 = 0.8 \text{ MPa}$$

$$T_2 = 24^\circ \text{C}$$

$$T_2 < T_{\text{sat}} \Rightarrow \text{Comp. Liquid}$$

by using sat. liquid line (A-14)

$$\text{at } T = 24^\circ \text{C}$$

$$v_2 = v_f = 0.0008761 \text{ m}^3/\text{kg}$$

$$u_2 = u_f = 84.44 \text{ kJ/kg}$$

$$s_2 = s_f = 0.31958 \text{ kJ/kg}\cdot\text{K}$$

$$\Delta s)_{\text{sys}} = s_2 - s_1 = -0.660 \text{ kJ/kg}\cdot\text{K}$$

$$\Delta s)_{\text{sur}} = \frac{Q}{T_{\text{in}}} = \frac{201.59}{20+273} = 0.688$$

$$\Delta s)_{\text{tot}} = 0.03 > 0$$

irrev.

$$= m \times 0.03 = \text{kJ/K}$$

$$Q - W = \Delta u$$

$$-Q - W_b = \Delta u$$

$$\Rightarrow -Q - P(v_2 - v_1) = (u_2 - u_1)$$

$$Q = 201.59 \text{ kJ/kg}$$

gain

④

R-134a

$$\Delta Q = 0$$

Closed system

$$V = 0.01 \text{ m}^3$$

⇒ sat. vapor

$$P_1 = 0.8 \text{ MPa}$$

(reversible manner)

+ (adiabatic or insulation)

= isentropic process

$$\Delta S = 0$$

$$S_2 = S_1$$

$$P_2 = 0.4 \text{ MPa}$$

$$T_2 = ??$$

$$W = ?$$

Sol

State 1

$$P_1 = 0.8 \text{ MPa (sat. vapor)}$$

Table (A-12)

$$v_1 = v_g = 0.02562 \text{ m}^3/\text{kg}$$

$$u_1 = u_g = 246.79 \text{ kJ/kg}$$

$$s_1 = s_g = 0.91835 \text{ kJ/kg} \cdot \text{K}$$

State 2

$$P_2 = 0.4 \text{ MPa}$$

$$s_2 = 0.91835 \text{ kJ/kg} \cdot \text{K}$$

by using sat. R-134a (A-12)

$$P_2 > 0.4 \text{ MPa}$$

$$s_f = 0.24 \quad s_f < s_2 < s_g$$

$$s_g = 0.92$$

* sat. mixture zone

$$X = \frac{s_2 - s_f}{s_g - s_f} = 0.98$$

$$u_2 = u_f + X u_{fg} = 231.84 \text{ kJ/kg}$$

$$T_2 = T_{\text{sat}} = 8.91^\circ\text{C}$$

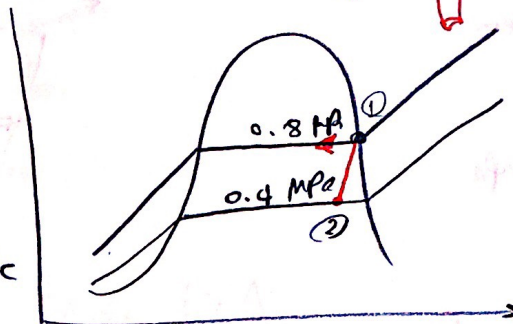
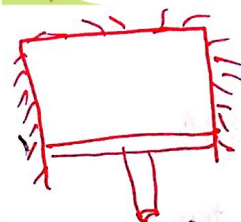


$$-W_b = \Delta u$$

$$-(W_b) = u_2 - u_1$$

$$W_b = 15.05 \text{ kJ/kg}$$

$$W_b = \frac{V}{M} \times 15.05 = \text{kJ}$$



6

19

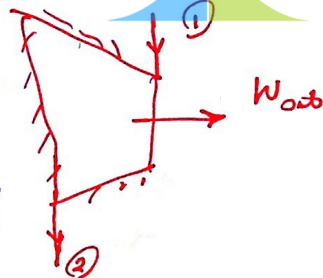
Steam
adiabatic

$\Delta Q = 0$
Open system $\rightarrow (h)$

$P_1 = 5 \text{ MPa}, T_1 = 400^\circ\text{C}$

$P_2 = 200 \text{ kPa}$

Rev. + adiab = isentropic



$W_{max} = ?$

$$-W = \Delta h + \cancel{\Delta K.E} + \cancel{\Delta P.E}$$

$$W = h_1 - h_2$$

State 1

$P_1 = 5 \text{ MPa}$

$T_1 = 400^\circ\text{C}$

② $P_1 = 5 \text{ MPa}, T_{sat} = 263^\circ\text{C}$

$T_1 > T_{sat}$ (A-5)

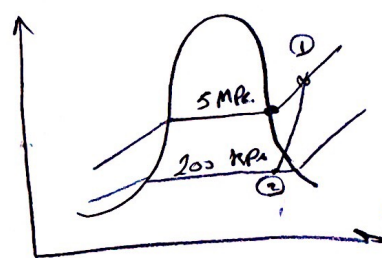
superheat (A-6)

$P_1 = 5 \text{ MPa}$

$T_1 = 400^\circ\text{C}$

$h_1 = 3196.7 \text{ kJ/kg}$

$s_1 = 6.64 \text{ kJ/kg}\cdot\text{K}$



State 2

$P_2 = 200 \text{ kPa}$

$s_2 = 6.64 \text{ kJ/kg}\cdot\text{K}$

Sat. steam (A-5)

$s_f = 1.53, s_g = 7.12$

$s_f < s_2 < s_g$

mix. rule

$$X = \frac{s_2 - s_f}{s_g - s_f} = 0.917$$

$$h_2 = h_f + X h_{fg} = 2508 \text{ kJ/kg}$$

$$W = 3196.7 - 2508 = 688.7 \text{ kJ/kg}$$



change of entropy for ideal Gases \Rightarrow

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مطلوب

From T-ds equations

$$\Delta S = \int_1^2 ds = \int_1^2 \frac{du}{T} + \int_1^2 \frac{pdv}{T} \quad (1)$$

$$\Delta S = \int_1^2 ds = \int_1^2 \frac{dh}{T} - \int_1^2 \frac{vdp}{T} \quad (2)$$

for ideal gas only

$$du = C_v(T) \cdot dT \quad \& \quad pV = RT$$

$$dh = C_p(T) \cdot dT$$

$$\Delta S = \int_1^2 \frac{C_v(T) \cdot dT}{T} + \int_1^2 R \frac{dv}{v} \quad (1)$$

$$\Delta S = \int_1^2 \frac{C_p(T) \cdot dT}{T} - \int_1^2 R \frac{dp}{p} \quad (2)$$

$$\Delta S = R \ln\left(\frac{V_2}{V_1}\right) + \int_{T_1}^{T_2} \frac{C_v(T) \cdot dT}{T}$$

$$\Delta S = \int_{T_1}^{T_2} \frac{C_p(T) \cdot dT}{T} - R \ln\left(\frac{P_2}{P_1}\right)$$

[1] Using constant specific heat method \Rightarrow

we will consider the specific heat are const. and equal their value at average Temp.

Table A.1
A.2

$$\Rightarrow \Delta S = C_{v_{av}} \cdot \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{V_2}{V_1}\right) \quad (\text{kJ/kg} \cdot \text{K})$$

$$\Rightarrow \Delta S = C_{p_{av}} \cdot \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right) \quad (\text{kJ/kg} \cdot \text{K})$$



(21)

where

$$* C_{V_{av}} = C_v @ T_{av} = \frac{T_1 + T_2}{2}$$

$$* C_{P_{av}} = C_p @ T_{av} = \frac{T_1 + T_2}{2}$$

OR Per unit mole \Rightarrow

$$\Rightarrow \bar{\Delta s} = \bar{C}_{V_{av}} \cdot \ln\left(\frac{T_2}{T_1}\right) + R_u \cdot \ln\left(\frac{V_2}{V_1}\right) \quad (\text{kJ/k.mole} \cdot \text{K})$$

$$\Rightarrow \bar{\Delta s} = \bar{C}_{P_{av}} \cdot \ln\left(\frac{T_2}{T_1}\right) - R_u \cdot \ln\left(\frac{P_2}{P_1}\right) \quad (\text{kJ/k.mole} \cdot \text{K})$$

$$\Delta s = \frac{\bar{\Delta s}}{M} \leftarrow \begin{array}{l} \text{molecular weight} \\ \text{molar mass} \end{array} \quad \text{Table. A.1}$$

$$* R_u = 8.314$$

$$* C_v = \frac{\bar{C}_v}{M}, \quad C_p = \frac{\bar{C}_p}{M}$$

Ideal gas \Rightarrow To convert from $(P \rightleftharpoons V)$

$$\begin{array}{l} P_2 V_2 = R T_2 \\ P_1 V_1 = R T_1 \end{array} \quad , \quad \overset{\text{air}}{R} = 0.287 \quad \text{kJ/kg} \cdot \text{K}$$

$$\frac{P_2 V_2}{P_1 V_1} = \frac{T_2}{T_1}$$



Thermo

الأحرار

23/3

ΔS for ideal gases.

[2] Using variable specific heat method \Rightarrow

$$\Delta S = \int_{T_1}^{T_2} \frac{C_p(T) \cdot dT}{T} - R \ln \left(\frac{P_2}{P_1} \right)$$

we will use the exact function of $C_p(T)$

$$\int_{T_1}^{T_2} \frac{C_p(T) \cdot dT}{T} = \int_0^{T_2} \frac{C_p(T) \cdot dT}{T} - \int_0^{T_1} \frac{C_p(T) \cdot dT}{T}$$

$$= \bar{S}_2 - \bar{S}_1$$

Ideal gas Tables.

where

$$S^0 = \int_0^T \frac{C_p(T) \cdot dT}{T}$$

Table (A-17)

T	u	h	S ⁰

$$\Rightarrow \Delta S = (\bar{S}_2 - \bar{S}_1) - R \ln \left(\frac{P_2}{P_1} \right) \quad (\text{kJ/kg} \cdot \text{K})$$

OR per unit mole

$$\bar{\Delta S} = (\bar{S}_2^0 - \bar{S}_1^0) - R_u \ln \left(\frac{P_2}{P_1} \right) \quad (\text{kJ/kmole} \cdot \text{K})$$

$$\Delta S = \frac{\bar{\Delta S}}{M} \quad (\text{kJ/kg} \cdot \text{K})$$

where

$$u = \int_0^T C_v(T) \cdot dT$$

$$h = \int_0^T C_p(T) \cdot dT$$

$$S^0 = \int_0^T \frac{C_p(T) \cdot dT}{T}$$



(Rev + adiabatic)
Isentropic process for Ideal Gases \Rightarrow

(23)

(A) Using Const specific heat method \Rightarrow

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$$\# \Delta S = C_{vav} \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{V_2}{V_1}\right) = 0$$

$$C_{vav} \ln\left(\frac{T_2}{T_1}\right) = -R \ln\left(\frac{V_2}{V_1}\right)$$

$$\ln\left(\frac{T_2}{T_1}\right) = \frac{-R}{C_{vav}} \ln\left(\frac{V_2}{V_1}\right)$$

$$\ln\left(\frac{T_2}{T_1}\right) = \ln\left(\frac{V_2}{V_1}\right)^{\frac{-R}{C_{vav}}} = \ln\left(\frac{V_1}{V_2}\right)^{\frac{R}{C_{vav}}}$$

$$R = C_{pav} - C_{vav}$$

$$\frac{R}{C_{vav}} = \left(\frac{C_{pav}}{C_{vav}}\right) - 1 = k_a - 1$$

$$\therefore \boxed{\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{(k_a - 1)}} \Rightarrow (1)$$

$$\# \Delta S = C_{pav} \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right) = 0$$

$$C_{pav} \ln\left(\frac{T_2}{T_1}\right) = R \ln\left(\frac{P_2}{P_1}\right)$$

$$\ln\left(\frac{T_2}{T_1}\right) = \ln\left(\frac{P_2}{P_1}\right)^{\left(\frac{R}{C_{pav}}\right)}$$

$$\therefore \boxed{\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\left(\frac{k_a - 1}{k_a}\right)}} \Rightarrow (2)$$

$$R = C_{pav} - C_{vav}$$

$$\frac{R}{C_{pav}} = 1 - \frac{C_{vav}}{C_{pav}} = 1 - \left(\frac{1}{k_a}\right)$$

$$1 - \frac{1}{k_a} = \frac{k_a - 1}{k_a}$$

Rigid Tank
adiabatic process

$$\Delta Q = 0$$

$$Q_2 - Q_1 = \Delta U = 0$$

$$U_2 = U_1$$

$$\therefore T_2 = T_1$$



From ① & ②

$$\boxed{\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k_a-1}{k}} = \left(\frac{V_1}{V_2}\right)^{k_a-1}}$$

(24)

Condition for using This equation :-

- 1- Ideal Gas.
- 2- Isentropic process.
- 3- Const. specific heat method.

from This equation \Rightarrow

$$\left(\frac{P_2}{P_1}\right)^{\frac{k_a-1}{k}} = \left(\frac{V_1}{V_2}\right)^{k_a-1}$$

$$\boxed{\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{k_a}} \Rightarrow \textcircled{3}$$

from equation ③

$$P_2 V_2^{k_a} = P_1 V_1^{k_a}$$

So, $\boxed{P V^k = C} \Rightarrow \text{Isentropic process.}$

Ahmed Kasem



(25)

② using Variable specific heat method \Rightarrow

$$\Delta S = (\dot{S}_2 - \dot{S}_1) - R \ln\left(\frac{P_2}{P_1}\right) = 0$$

$$\dot{S}_2 - \dot{S}_1 = R \ln\left(\frac{P_2}{P_1}\right)$$

$$\ln\left(\frac{P_2}{P_1}\right) = \frac{\dot{S}_2}{R} - \frac{\dot{S}_1}{R}$$

$$\frac{P_2}{P_1} = \exp\left(\frac{\dot{S}_2}{R} - \frac{\dot{S}_1}{R}\right)$$

$$\Rightarrow \frac{P_2}{P_1} = \frac{\exp(\dot{S}_2/R)}{\exp(\dot{S}_1/R)}$$

u	h	s°	Pr

with $T_1 \Rightarrow$ get P_{r1}
from $\frac{P_{r2}}{P_{r1}} = \frac{P_2}{P_1} \Rightarrow$ get $P_{r2} = \checkmark$
with $P_{r2} \Rightarrow$ get (T_2)

$$\therefore \boxed{\frac{P_2}{P_1} = \frac{P_{r2}}{P_{r1}}} \rightarrow \text{where } P_r = \exp(\dot{S}/R) \quad \text{①}$$

from \therefore

$$P_2 v_2 = R T_2$$

$$P_1 v_1 = R T_1$$

$$\frac{P_2 v_2}{P_1 v_1} = \frac{T_2}{T_1}$$

$$\frac{v_2}{v_1} = \frac{T_2}{T_1} \times \frac{P_1}{P_2}$$

$$\frac{v_2}{v_1} = \frac{T_2}{T_1} \times \frac{P_{r1}}{P_{r2}}$$

$$\frac{v_2}{v_1} = \frac{(T_2/P_{r2})}{(T_1/P_{r1})}$$

$$\therefore \boxed{\frac{v_2}{v_1} = \frac{v_{r2}}{v_{r1}}} \rightarrow \text{②}$$

where $v_r = \frac{T_{\text{Kelvin}}}{P_r}$

Condition for using these equation ① & ②

1. Ideal Gas.
2. Isentropic Process.
3. variable specific heat method.

with $T_1 \Rightarrow$ get v_{r1}

from ② \Rightarrow get $v_{r2} = \checkmark$

with $v_{r2} \Rightarrow$ get (T_2)



Ex

adiabatic system

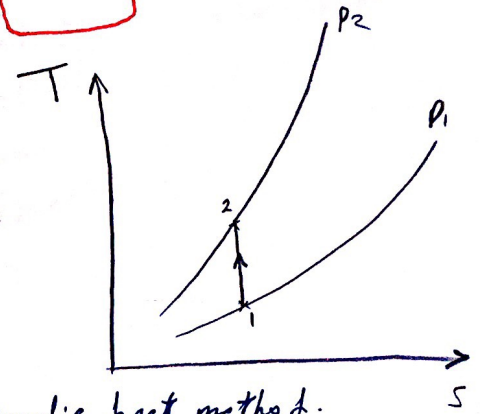
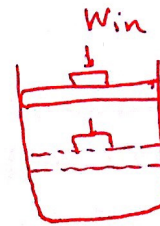
$$P_1 = 95 \text{ kPa}$$

$$T_1 = 22^\circ\text{C} + 273 = 295 \text{ K}$$

rev. process.

$$\text{Compression ratio} = \frac{V_1}{V_2} = 8$$

$$\text{Req. } T_2 \text{ \& } W_{in}$$



(26)

Sol

① using variable specific heat method

From ideal gas Table Air

$$T_1 = 295 \text{ K} \quad \text{Table (A-17)}$$

$$v_{r1} = 647.9$$

$$u_1 = 210.49 \text{ kJ/kg}$$

$$\frac{v_{r2}}{v_{r1}} = \frac{v_2}{v_1} = \frac{1}{8}$$

$$v_{r2} = 80.99$$

$$u = v_r$$

$$481.01 \quad 81.89$$

$$u_2 \quad 80.99$$

$$488.8 \quad 78.61$$

$$\frac{u_2 - 481.01}{488.8 - 481.01} = \frac{80.99 - 81.89}{78.61 - 81.89}$$

$$u_2 = 483.14 \text{ kJ/kg}$$

1st law

$$Q_{12} - W_{12} = u_2 - u_1$$

$$W_{12} = u_1 - u_2 = -272.6$$

② Const specific heat method.

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{k-1}$$

$$k @ T_1 = 1.4$$

$$T_2 = 295 \times (8)^{0.4} = 677.73 \text{ K}$$

$$T_{av} = \frac{T_1 + T_2}{2} = 486.4 \text{ K}$$

$$k_{av @ T_{av}} = 1.39$$

$$T_2 = 663.78 \text{ K}$$

$$Q_{12} - W_{12} = u_2 - u_1$$

$$W = u_1 - u_2 = C_{v,av} (T_1 - T_2)$$

$$210.4 - 483.14 = -C_v$$

$$= 0.733 (T_1 - T_2)$$

$$= -270.25 \text{ kJ/kg}$$

(9)

N_2

$m = 1.2 \text{ kg}$

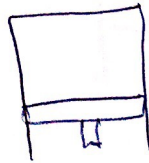
$P_1 = 120 \text{ kPa}$

$T_1 = 27 + 273 = 300 \text{ K}$

$PV^{1.3} = \text{const}$

$V_2 = \frac{1}{2} V_1$

Polytropic



① Isothermal

② Adiabatic

③ Isentropic

④ Polytropic

⑤ Isobaric

⑥ Isochoric

ideal state

$\Delta S = ?$

Sol

Table A.1. & ②

* by using Const. specific heat method.

(closed system)

$$\Delta S = C_{var} \ln \frac{T_2}{T_1} + R \ln \frac{V_2}{V_1}$$

⇒ ① Ideal Gas

② closed system

③ Const specific h

$$C_{var} \rightarrow T_{av} \approx PV^k = c$$

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2} \right)^{k_{av}}$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2} \right)^{\frac{k_{av}-1}{k_{av}}}$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2} \right)^{0.3}$$

$$T_2 = 369.34 \text{ K}$$

$$T_{av} = \frac{T_1 + T_2}{2} = 334.67 \text{ K}$$

$$C_{var} \xrightarrow{(A.2)} = \text{Interpolation} = 0.743 \text{ kJ/kg} \cdot \text{K}$$

$$\text{Table A.1} \rightarrow R = 0.2968 \text{ kJ/kg} \cdot \text{K}$$

$$\Delta S = 0.743 \text{ kJ/kg} \cdot \text{K} \ln \frac{369.34}{300} + 0.2968 \ln \frac{0.5}{1}$$

$$= -0.052 \text{ kJ/kg} \cdot \text{K}$$

$N_2 \rightarrow$ up to 1000 K
constant C_{var} is used



26.b

15

1st law

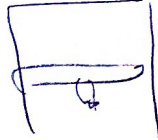
Air

$$P_1 = 90 \text{ kPa}$$

closed system

$$T_1 = 20 + 273 = 293 \text{ K}$$

$$P_2 = 400 \text{ kPa}$$



reversible + isothermal proc

$$\Delta S = \frac{Q}{T}$$

$$\Delta S = ?$$

$$W_{in} = ?$$

sol

$$\Delta S = C_{v,m} \ln \left(\frac{T_2}{T_1} \right) + R \ln \left(\frac{V_2}{V_1} \right)$$

$$\frac{P_2 V_2}{P_1 V_1} = \frac{P_1}{P_2}$$

$$\Delta S = C_p \ln \frac{T_2}{T_1} + R \ln \frac{P_2}{P_1}$$

$$\Delta S = -0.428 \text{ kJ/kg} \cdot \text{K}$$

$$Q - W = \Delta U = C_v \Delta T$$

$$Q = T \Delta S = -125.4 \text{ kJ/kg}$$

$$Q_{out} = 125.4 \text{ kJ/kg}$$

$$Q = W$$

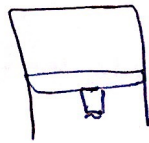


(12) Air

26.c

$$P_1 = 100 \text{ kPa}$$

$$T_1 = 17 + 273 = 290 \text{ K}$$



$$P_2 = 800 \text{ kPa}$$

reversible adiabatic process = isentropic

$$T_2 = ?$$

$$W = ? \quad \oint \delta w = \Delta u = C_{v,air} \Delta T$$

a) Const specific heat method:

b) Variable specific heat method.

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{k_{av} - 1}{k_{av}}}$$

$$k \text{ at } T_1 = 1.4 \text{ (from } \gamma = 3 \dots)$$

$$T_2 = 525.3 \text{ K}$$

$$T_{av} = \frac{T_1 + T_2}{2} =$$

$$k_{av} \text{ @ } T_{av} =$$

$$T_2 =$$

$$\oint \delta w = \Delta u = C_{v,air} \Delta T$$

$$W =$$

$$KJ/Kg$$

$$\frac{P_2}{P_1} = \frac{P_{r2}}{P_{r1}} \quad \frac{V_2}{V_1} = \frac{V_{r2}}{V_{r1}}$$

states within system

states within system

Table A-17

$$T_1 =$$

$$h_1 =$$

$$P_{r1} =$$

$$P_{r2} =$$

$$T_2 = 524.4$$

$$h_2 =$$

$$W =$$

$$KJ/Kg$$

reversible

u + w bon
جس



$$\Delta Q = 0$$

n_1

$$\sum F_{in} = \sum F_{out}$$

$$Q - W = \Delta h + \Delta k.E + \Delta P$$

$$k.E = \frac{V_2^2}{2} - \frac{V_1^2}{2}$$

$$0 = h_2 - h_1 =$$

$$h_1 - h_2 = \frac{V_2^2 - V_1^2}{2} \times \frac{1}{1000}$$

$$\frac{m^2}{s^2}$$

$$J/kg = \frac{N.m}{kg} = \frac{kg.m}{s^2} \cdot \frac{m}{kg} = m^2/s^2$$

$$N.m \Rightarrow J$$

$$Power = work/time$$

A-17

$$T_2 =$$

$$h_1 = r$$

$$h \neq r$$



Therm 6
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الأحرار

(27)

Ahmed
Kasem

* Reversible steady flow work \Rightarrow

The moving boundary work

$W_b = \int P dV$ is The work for (closed system) ..

\Rightarrow Similar expression can be obtained for steady flow device (open system) C.V

The 1st law for steady flow device under rev. process ..

$$\delta q_{rev} - \delta w_{rev} = dh + dk.e + dp.e$$

From 2nd law $\delta q_{rev} = T.ds$

$$T.ds \rightarrow \delta w_{rev} = dh + dk.e + dp.e$$

From T.ds relations

$$T.ds = dh - v.dp$$

$$\therefore dh - v.dp - \delta w_{rev} = dh + dk.e + dp.e$$

$$\delta w_{rev} = -v.dp - dk.e - dp.e$$

$$W_{rev} = \int_1^2 -v.dp - \int_1^2 dk.e - \int_1^2 dp.e$$

$$W_{rev} = \int_1^2 -v.dp - \Delta k.e - \Delta p.e$$

if $\Delta k.e = 0$
 $\Delta p.e = 0$

$$W_{rev} = \int -v.dp \Rightarrow$$



From This equation we find That \Rightarrow

(28)

- 1- The integration can be performed if

$$v = f(p) \quad \Leftarrow \text{is given}$$
- 2- This equation is valid for the work given to device (Pump or Compressor)
 OR The work given by the device (Turbine)
- 3- The steady flow work mainly depends on the specific volume (v)
 such that:-

a) For Incompressible fluid $p = c, v = c$

$$W_{rev} = \int_1^2 -v dp$$

$$= -v \int_1^2 dp$$

$$W_{rev} = -v(p_2 - p_1)$$

$$W_{rev} = v(p_1 - p_2) \quad \text{KJ/kg} \quad \Rightarrow \text{Work Pump (open system)}$$

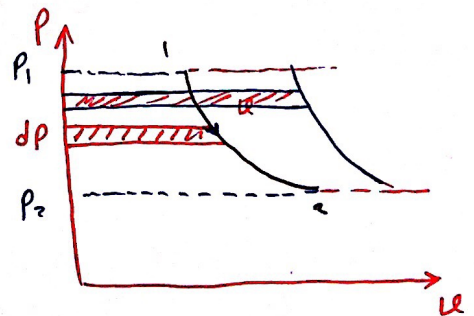
Work Pump \Rightarrow it is a reversible work given to a pump to raise the liquid pressure from (p_1) to (p_2)

b)

as shown the reversible steady flow is the area to the left of the process curve on P.v diagram

also, it is shown that

The larger the specific volume, the larger the reversible steady flow work.
 and the smaller, the smaller



So,



it is recommended to keep the specific volume as small as possible during the compression process. (Compressor)

& as large as possible during the expansion process. (Turbine)

Ex

Determine the work required to compress water from 100 kPa to 1 MPa isentropically. if

- ① water is sat. liquid
- ② water is sat vapor

Sol

Through the process (1-2), all states are compressed liquid (incompressible fluid) must be pump

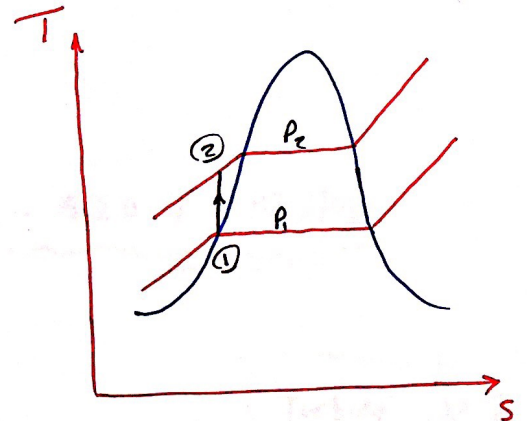
$$W = \int -v dp, \quad v = C$$

$$W = v(p_1 - p_2)$$

$$v_1 = v_f @ p_1$$

$$v_1 = v_f = \frac{0.0010403}{0.001043} \text{ m}^3/\text{kg}$$

$$\therefore W = 0.0010403 (100 - 1000) = \underline{\underline{-0.94 \text{ kJ/kg}}}$$



[2]



(30)

all states (1-2) are in vapour phase (Compressor)

$$W = \int -v dp \Rightarrow \text{لأننا نستخدم تلك العلاقة}$$

$v \neq C$ لأن

1st law

$$q_{1,2} - W_{1,2} = h_2 - h_1 + \Delta k.e + \Delta p.e$$

$$W_{1,2} = h_1 - h_2$$

state. 1

$$h_1 = h_g @ P_1 \text{ from Table (A-5)}$$

$$h_1 = 2675 \text{ kJ/kg}$$

$$s_1 = s_g = 7.3589 \text{ kJ/kg} \cdot K$$

state. 2

$$s_2 = s_1 = 7.3589 \text{ kJ/kg} \cdot K$$

$$P_2 = 1000 \text{ kPa}$$

$$s_p =$$

$$s_g = 6.585$$

$$s_2 > s_g$$

at super heat

Table (A-6)

$$P_2 = 1 \text{ MPa}$$

$$s_2 = 7.3589 \text{ kJ/kg} \cdot K$$

$$3158.2 \quad 7.3029$$

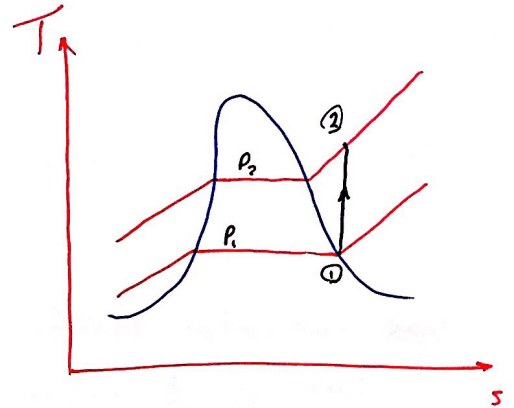
$$h_2 \quad 7.3589$$

$$3264.5 \quad 7.4670$$

$$h_2 = 3195.45 \text{ kJ/kg}$$

$$W = h_1 - h_2 = -520.4 \text{ kJ/kg}$$

Turbine ←



Thermo

3/4



minimizing The Compressor work \Rightarrow

(31)

$$w = \int -v dp$$

$$Pv = RT$$

$$v = R \cdot \frac{T}{P}$$

To minimize The work of compressor, The specific volume must kept as small as possible, This can be done by making The gas Temp. as low as possible and This can be achieved practically by cooling gas as it's compressed.

To understand The effect of cooling during The compression process
We will determine work reversible w_{rev} for \Rightarrow

- ① Isentropic process (adiabatic, no cooling), $Pv^k = C$
- ② Polytropic process (some cooling), $Pv^n = C$, $1 < n < k$
- ③ Isothermal process (maximum cooling), $Pv = C$

Ahmet

Kasem



1 Isentropic process \Rightarrow

$$W_{rev} = \int -v dp$$

$$Pv^k = c$$

$$v = \left(\frac{c}{P}\right)^{\frac{1}{k}}$$

$$W_{rev} = \int - \left(\frac{c}{P}\right)^{\frac{1}{k}} dP$$

$$= - (c)^{\frac{1}{k}} \int P^{-\frac{1}{k}} dP$$

$$= - \frac{k}{k-1} (c)^{\frac{1}{k}} P^{\frac{-\frac{1}{k}+1}{1}} \Big|_1^2$$

$$W_{rev} = - \frac{k}{k-1} (c)^{\frac{1}{k}} \left[P_2^{\frac{k-1}{k}} - P_1^{\frac{k-1}{k}} \right]$$

$$= \frac{k}{k-1} (c)^{\frac{1}{k}} \left[P_1^{\frac{k-1}{k}} - P_2^{\frac{k-1}{k}} \right]$$

$$Pv^k = c$$

$$P^{\frac{1}{k}} v = c^{\frac{1}{k}} = P_1^{\frac{1}{k}} v_1 = P_2^{\frac{1}{k}} v_2$$

$$1) W_{rev} = \frac{k}{k-1} \left[P_1^{\frac{k-1}{k}} P_1^{\frac{1}{k}} v_1 - P_2^{\frac{k-1}{k}} P_2^{\frac{1}{k}} v_2 \right]$$

$$2) = \frac{k}{k-1} \left[P_1 v_1 - P_2 v_2 \right]$$

$$3) = \frac{k}{k-1} \left[RT_1 - RT_2 \right]$$

$$= \frac{kRT_1}{k-1} \left[1 - \frac{T_2}{T_1} \right]$$

$$W_{rev} = \frac{kRT_1}{k-1} \left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} \right]$$

KJ/kg

2 Polytropic process \Rightarrow 32

$$W_{rev} = \int -v dp$$

$$Pv^n = c$$

$$v = \left(\frac{c}{P}\right)^{\frac{1}{n}}$$

$$W_{rev} = \int - \left(\frac{c}{P}\right)^{\frac{1}{n}} dP$$

$$= - (c)^{\frac{1}{n}} \int P^{-\frac{1}{n}} dP$$

$$= - \frac{n}{n-1} (c)^{\frac{1}{n}} P^{\frac{-\frac{1}{n}+1}{1}} \Big|_1^2$$

$$W_{rev} = \frac{n}{n-1} (c)^{\frac{1}{n}} \left[P_1^{\frac{n-1}{n}} - P_2^{\frac{n-1}{n}} \right]$$

$$(c)^{\frac{1}{n}} = P_1^{\frac{1}{n}} v_1 = P_2^{\frac{1}{n}} v_2$$

$$W_{rev} = \frac{n}{n-1} \left[P_1 v_1 - P_2 v_2 \right]$$

$$= \frac{n}{n-1} \left[RT_1 - RT_2 \right]$$

$$= \frac{nRT_1}{n-1} \left[1 - \frac{T_2}{T_1} \right]$$

$$W_{rev} = \frac{nRT_1}{n-1} \left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} \right] \text{ KJ/kg}$$



2 Isothermal Process \Rightarrow

33

$$W_{rev} = \int -v dp$$

$$pv = c$$

$$v = \frac{c}{p}$$

$$W_{rev} = - \int \left(\frac{c}{p} \right) dp$$

$$= - (c) \int \frac{dp}{p}$$

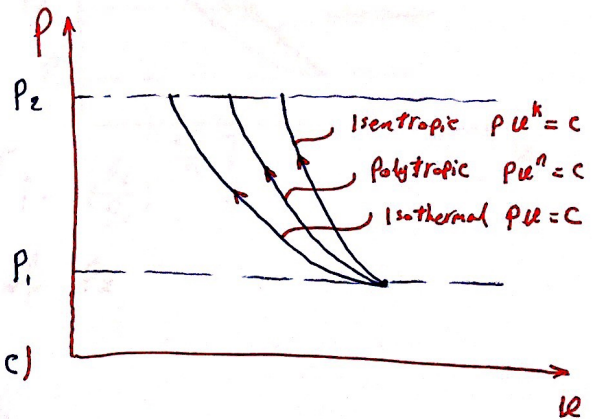
$$= - (c) \ln \frac{p_2}{p_1}$$

$$= - RT_1 \ln \frac{p_2}{p_1}$$

$$W_{rev} = RT_1 \ln \left(\frac{p_1}{p_2} \right) \text{ kJ/kg}$$

The 3 processes are shown on P-v diagram \Rightarrow

- ① The adiabatic process ($pv^k = c$) requires the maximum work
- ② The isothermal process ($pv = c$) requires the minimum work.
- ③ The work for polytropic process ($pv^n = c$) is in between.



$$n \downarrow \Rightarrow w \downarrow$$

by increasing heat rejection during the compression process.



* Multistage Compression with Intercooling \Rightarrow

(34)

Ideal Two-stage Compression

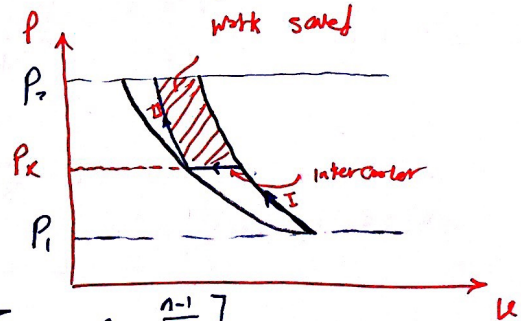
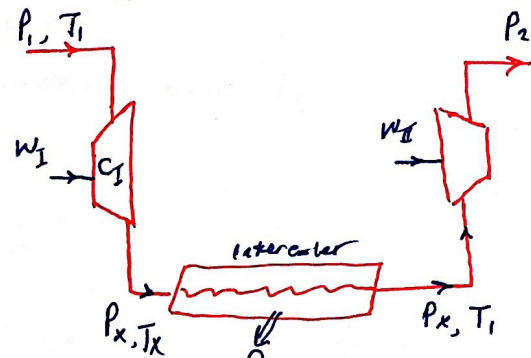
The gas is compressed in stages and cooled between each stage by using heat exchanger (intercooler).

Ideally, the gas is cooled to its initial Temp. (T_1), Through the Const. Pressure.

as shown. The heating area represent the (saved work)

$$\text{Saved Work} = W_{\text{one stage}} - W_{\text{2 stage}}$$

2 stages



$$W_{\text{comp}} = W_I + W_{II}$$

$$= \frac{nRT_1}{n-1} \left[1 - \left(\frac{P_x}{P_1} \right)^{\frac{n-1}{n}} \right] + \frac{nRT_1}{n-1} \left[1 - \left(\frac{P_2}{P_x} \right)^{\frac{n-1}{n}} \right]$$

To obtain the optimum value of P_x

$$\frac{dW_{\text{comp}}}{dP_x} = 0$$

$$P_x = \sqrt{P_1 \cdot P_2}$$

OR

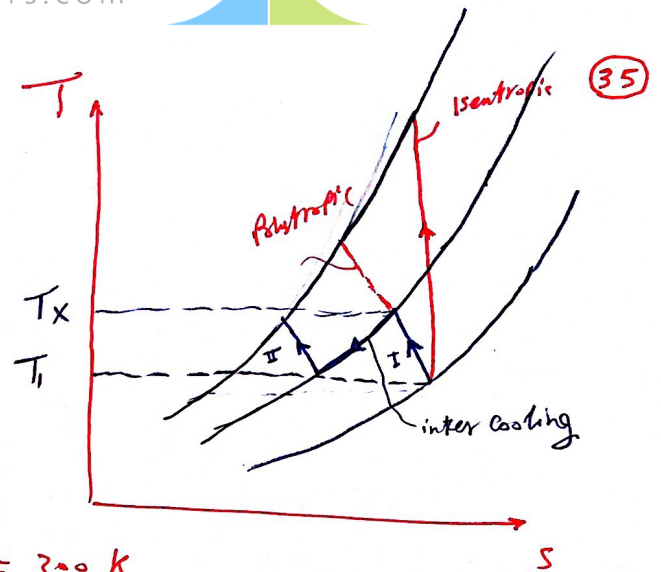
$$\frac{P_x}{P_1} = \frac{P_2}{P_x}$$

This mean that

$$W_I = W_{II}$$

SO

$$W_{\text{comp}} = \frac{2nRT}{n-1} \left[1 - \left(\frac{P_x}{P_1} \right)^{\frac{n-1}{n}} \right]$$



Air is compressed by reversible compressor from $P_1 = 100 \text{ kPa}$, $T_1 = 300 \text{ K}$

① Isentropic ~~gas~~ Compressor. with $k=1.4$

$$W = \frac{k R T_1}{k-1} \left[1 - \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}} \right]$$

$$= \frac{1.4 \times 0.287 \times 300}{1.4 - 1} \left[1 - \left(\frac{900}{100} \right)^{\frac{1.4-1}{1.4}} \right] = -263.2 \text{ kJ/kg}$$

$$W = \frac{nRT_1}{n-1} \left[1 - \left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} \right] = -246.4 \text{ kJ/kg}$$
$$W = RT_1 \ln\left(\frac{P_2}{P_1}\right) = (0.287) \times 300 \ln\left(\frac{100}{900}\right) = -189.2 \text{ kJ/kg}$$
$$W = \frac{2nRT_1}{n-1} \left[1 - \left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} \right]$$

$$P_K = \sqrt{P_1 \cdot P_2} = 300$$

$$W = -215.3 \text{ kJ/kg}$$